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COMPUTER PROGRAMS FOR ANTENNA PATTERN SYNTHESIS

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13. ABSTRACT

This report contains computer programs, instructions, and sample input-output data for antenna pattern synthesis as developed in the previous report "Computational Methods for Antenna Pattern Synthesis." The programs are valid for point sources arbitrarily distributed in a plane, and for pattern synthesis in this plane. Included are programs for synthesis with (1) pattern magnitude and phase specified, (2) pattern magnitude only specified, (3) these two cases with a constraint on the source norm, and the first two cases with a constraint on the source quality factor. Also included are programs to compute and plot the specified and synthesized patterns.

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ABSTRACT

This report contains computer programs, instructions, and sample input-output data for antenna pattern synthesis as developed in the previous report "Computational Methods for Antenna Pattern Synthesis." The programs are valid for point sources arbitrarily distributed in a plane, and for pattern synthesis in this plane. Included are programs for synthesis with (1) pattern magnitude and phase specified, (2) pattern magnitude only specified, (3) these two cases with a constraint on the source norm, and (4) the first two cases with a constraint on the source quality factor. Also included are programs to compute and plot the specified and synthesized patterns.

CONTENTS

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I. INTRODUCTION

The programs used to compute the examples of Scientific Report No. 2, entitled "Computational Methods for Antenna Pattern Synthesis," are described and listed in this report. Each program is accompanied by an explanation of the input data, a verbal flow chart, and sample input-output data. Equations of the previous report [1] will be denoted (2-), and equations of this report will be denoted by a single number.

The programs are written to be valid for N point sources arbitrarily distributed in a plane, and for pattern synthesis in this plane. However, for the examples all synthesized patterns are radiated by 10 point sources equally spaced on the half of an ellipse on one side of the minor axis. The major axis of the ellipse is twice as long as the minor axis. The first and last point sources are at the ends of the minor axis. Pattern synthesis is accomplished by adjusting the complex amplitudes of the 10 point sources so that the pattern radiated by these point sources is as close as possible to a specified pattern. The amplitude of the specified pattern used is sketched below.

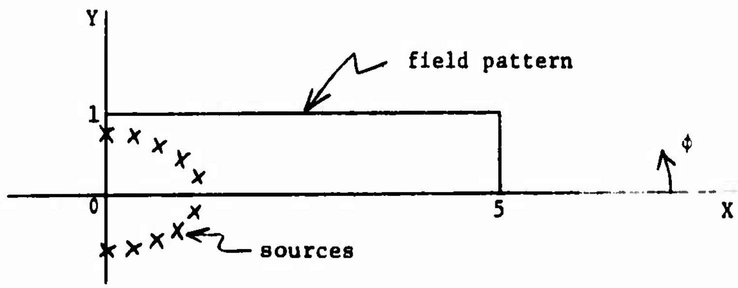

Figure 1. 10 point sources and specified pattern.

[1] J. R. Mautz and R. F. Harrington, "Computational Methods for Pattern Synthesis," Scientific Report No. 2 on Contract F19628-73-C-0047 between Air Force Cambridge Research Laboratory and Syracuse University.

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II. PATTERN SYNTHESIS

The program of this section computes the vector $\vec{f}$ of point source strengths given by (2-16). This $\vec{f}$ is the result of pattern synthesis with phase specified. The program prints the magnitudes and phases of the elements of $\vec{f}$. The program also punches on cards the real and imaginary parts of the elements of $\vec{f}$ for input into the program of section IX of this report.

Punched card data is read into the program according to

```
      READ(1,30) N,M,NK,NG
30    FORMAT(4I3)
      READ(1,33)(BK(I), I=1, NK)
33    FORMAT(5E14.7)
      READ(1,10)(X(I), I=1, N)
10    FORMAT(10F8.4)
      READ(1,10)(Y(I), I=1, N)
      READ(1,10)(PHI(I), I=1, M)
      DO 35 JK=1, NK
      DO 36 JG=1, NG
      READ(1,10)(G(I), I=1, M)
36    CONTINUE
35    CONTINUE
```

The x and y coordinates of N point sources are read in as X and Y. The specified pattern $\vec{g}_o$ of (2-16) is read in as G at M angles PHI (denoted by ϕ in Figure 1) in degrees in the plane of the point sources. The index JK of DO loop 35 denotes the JKth propagation constant BK(JK). DO loop 36 makes it possible to deal successively with NG different specified patterns at the same propagation constant BK(JK).

Minimum allocations are given by

```
      COMPLEX C(N*N)
      DIMENSION LR(N)
```

in the subroutine LINEQ and by

```
COMPLEX G(M), T(N*M), TC(N*M)
     TT(N*N), TG(N), F(N), TF(M)
DIMENSION BK(NK), X(N), Y(N), PHI(M),
     TFM(M), FM(N), FP(N)
```

in the main program.

Nested DO loops 14 and 15 store the matrix element given by (2-18) in T((m-1)*N+n) and its complex conjugate in TC((m-1)*N+n). Assuming that [W] is the identity matrix, nested DO loops 16 and 17 store the matrix $[\tilde{T}^*T]$ of (2-16) by columns in TT. The subroutine LINEQ inverts the N by N matrix stored in TT. Nested DO loops 19 and 20 store $[\tilde{T}^*]\vec{g}_o$ of (2-16) in TG. DO loop 22 stores $\vec{f}$ of (2-16) in F and, assuming that [V] appearing in (2-9) is the identity matrix, $\|\vec{f}\|^2$ in FF. Do loop 22 discovers that the element of $\vec{f}$ largest in magnitude is stored in F(J2).

DO loop 24 stores the complex synthesized pattern $[T]\vec{f}$ of (2-17) in TF and the magnitude of the synthesized pattern in TFM. $\|[T]\vec{f}\|^2$ is accumulated in TFS, the pattern synthesis error $\|[T]\vec{f} - \vec{g}_o\|^2$ in E, and $\|\vec{g}_o\|^2$ in GG. Just after DO loop 24, Q of (2-15) is stored in Q and E is normalized by dividing by GG. Statement 59 punches F on cards for possible input into the program of section IX. DO loop 41 normalizes the element of $\vec{f}$ largest in magnitude to unity and then stores the magnitudes and phases in degrees of $\vec{f}$ in FM and FP respectively.

The propagation constant BK of the input data is such that the point sources are spaced about a quarter of a wavelength apart on the ellipse. Actually, BK, X, and Y were calculated by a short auxiliary computer program not listed in this report. In the sample input data the magnitude of the specified pattern G is given in Figure 1, G is real and G is evaluated at $\phi = 5°, 15°, 25°, \ldots 355°$ in the plane of the ellipse. The numbers printed out under the headings FM and FP in the sample output were used to construct the first two columns of Table 1 of Report No. 2.

LISTING OF PATTERN SYNTHESIS PROGRAM

```
//                 (0034,FF,15S,1,4),'MAUTZ,JOE',REGION=200K
// EXEC WATFIV
//GO.FT02F001 DD SYSOUT=B,DCB=(RECFM=F,BLKSIZE=80)
//GO.SYSIN DD *
$JOB               MAUTZ,TIME=1,PAGES=40
      SUBROUTINE LINEQ(LL,C)
      COMPLEX C(100),STOR,STO,ST,S
      DIMENSION LR(50)
      DO 20 I=1,LL
      LR(I)=I
   20 CONTINUE
      M1=0
      DO 18 M=1,LL
      K=M
      DO 2 I=M,LL
      K1=M1+I
      K2=M1+K
      IF(ABS(REAL(C(K1)))+ABS(AIMAG(C(K1)))-ABS(REAL(C(K2)))-ABS(AIMAG(C
     1(K2)))) 2,2,6
    6 K=I
    2 CONTINUE
      LS=LR(M)
      LR(M)=LR(K)
      LR(K)=LS
      K2=M1+K
      STOR=1./C(K2)
      J1=0
      DO 7 J=1,LL
      K1=J1+K
      K2=J1+M
      STO=C(K1)
      C(K1)=C(K2)
      C(K2)=STO*STOR
      J1=J1+LL
    7 CONTINUE
      K1=M1+M
      C(K1)=STOR
      DO 11 I=1,LL
      IF(I-M) 12,11,12
   12 K1=M1+I
      ST=C(K1)
      C(K1)=0.
      J1=0
      DO 10 J=1,LL
      K1=J1+I
      K2=J1+M
      C(K1)=C(K1)-C(K2)*ST
      J1=J1+LL
   10 CONTINUE
   11 CONTINUE
      M1=M1+LL
   18 CONTINUE
      J1=0
      DO 9 J=1,LL
      IF(J-LR(J)) 14,8,14
   14 LRJ=LR(J)
      J2=(LRJ-1)*LL
   21 DO 13 I=1,LL
```

```
      K2=J2+I
      K1=J1+I
      S=C(K2)
      C(K2)=C(K1)
      C(K1)=S
   13 CONTINUE
      LR(J)=LR(LRJ)
      LR(LRJ)=LRJ
      IF(J-LR(J)) 14,8,14
    8 J1=J1+LL
    9 CONTINUE
      RETURN
      END
      COMPLEX U,U1,CONJG,G(36),T(360),TC(360),TT(100),TG(10),F(10)
      COMPLEX TF(36)
      DIMENSION BK(10),X(10),Y(10),PHI(36),TFM(36),FM(10),FP(10)
      U=(0.,1.)
      PI=3.141593
      C=PI/180.
      READ(1,30) N,M,NK,NG
   30 FORMAT(4I3)
      WRITE(3,32) N,M,NK,NG
   32 FORMAT('0  N   M NK NG'/1X,4I3)
      READ(1,33)(BK(I),I=1,NK)
   33 FORMAT(5E14.7)
      WRITE(3,34)(BK(I),I=1,NK)
   34 FORMAT('0BK'/(1X,5E14.7))
      READ(1,10)(X(I),I=1,N)
   10 FORMAT(10F8.4)
      WRITE(3,13)(X(I),I=1,N)
   13 FORMAT('0X'/(1X,10F8.4))
      READ(1,10)(Y(I),I=1,N)
      WRITE(3,37)(Y(I),I=1,N)
   37 FORMAT('0Y'/(1X,10F8.4))
      READ(1,10)(PHI(I),I=1,M)
      WRITE(3,11)(PHI(I),I=1,M)
   11 FORMAT('0PHI'/(1X,10F8.4))
      ZM=M
      DO 35 JK=1,NK
      BB=BK(JK)
      J1=0
      DO 14 K=1,M
      S1=PHI(K)*C
      CS=COS(S1)
      SN=SIN(S1)
      DO 15 I=1,N
      J1=J1+1
      S1=BB*(X(I)*CS+Y(I)*SN)
      T(J1)=COS(S1)+U*SIN(S1)
      TC(J1)=CONJG(T(J1))
   15 CONTINUE
   14 CONTINUE
      J1=0
      DO 16 J=1,N
      DO 17 I=1,N
      J2=J
      J3=I
      J1=J1+1
      TT(J1)=0.
      DO 18 K=1,M
```

```
      TT(J1)=TT(J1)+TC(J3)*T(J2)
      J3=J3+N
      J2=J2+N
   18 CONTINUE
   17 CONTINUE
   16 CONTINUE
      CALL LINEQ(N,TT)
      DO 36 JG=1,NG
      READ(1,10)(G(I),I=1,M)
      WRITE(3,12)(G(I),I=1,M)
   12 FORMAT('0G'/(1X,10F8.4))
      DO 19 J=1,N
      J2=J
      TG(J)=0.
      DO 20 K=1,M
      TG(J)=TG(J)+TC(J2)*G(K)
      J2=J2+N
   20 CONTINUE
   19 CONTINUE
      FF=0.
      S2=0.
      DO 22 J=1,N
      F(J)=0.
      J1=J
      DO 23 I=1,N
      F(J)=F(J)+TT(J1)*TG(I)
      J1=J1+N
   23 CONTINUE
      S1=F(J)*CONJG(F(J))
      FF=FF+S1
      IF(S1.LT.S2) GO TO 22
      S2=S1
      J2=J
   22 CONTINUE
      J1=0
      TFS=0.
      E=0.
      GG=0.
      DO 24 K=1,M
      TF(K)=0.
      DO 25 I=1,N
      J1=J1+1
      TF(K)=TF(K)+T(J1)*F(I)
   25 CONTINUE
      S1=TF(K)*CONJG(TF(K))
      TFM(K)=SQRT(S1)
      TFS=TFS+S1
      U1=TF(K)-G(K)
      E=E+U1*CONJG(U1)
      GG=GG+G(K)*CONJG(G(K))
   24 CONTINUE
      Q=ZM*FF/TFS
      E=E/GG
      WRITE(3,26) GG,FF,E,Q
   26 FORMAT('0GG=',E14.7,' FF=',F14.7,' E=',E14.7,' Q=',E14.7)
      WRITE(3,27)(F(I),I=1,N)
   27 FORMAT('0F'/(1X,5F14.7))
   59 WRITE(2,38)(F(I),I=1,N)
   38 FORMAT(5F14.7)
      U1=1./F(J2)
```

```
      DO 41 J=1,N
      F(J)=U1*F(J)
      FM(J)=CABS(F(J))
      S1=REAL(F(J))
      S2=AIMAG(F(J))
      FP(J)=ATAN2(S2,S1)
      FP(J)=FP(J)/C
   41 CONTINUE
      WRITE(3,42)(FM(J),J=1,N)
   42 FORMAT('OFM'/(1X,10F7.3))
      WRITE(3,43)(FP(J),J=1,N)
   43 FORMAT('OFP'/(1X,10F7.1))
      WRITE(3,28)(TF(I),I=1,M)
   28 FORMAT('OTF'/(1X,10F8.4))
      WRITE(3,29)(TFM(I),I=1,M)
   29 FORMAT('OTFM'/(1X,10F8.4))
   36 CONTINUE
   35 CONTINUE
      STOP
      END
..$DATA
 10 36  1  1
 0.2908882E+00
  0.0000   5.3608  10.6048  15.5027  19.2648  19.2648  15.5027  10.6048   5.3608   0.0000
-10.0000  -9.6320  -8.4706  -6.2894  -2.5559   2.5559   6.2894   8.4706   9.6320  10.0000
  5.0000  15.0000  25.0000  35.0000  45.0000  55.0000  65.0000  75.0000  85.0000  95.0000
105.0000115.0000125.0000135.0000145.0000155.0000165.0000175.0000185.0000195.0000
205.0000215.0000225.0000235.0000245.0000255.0000265.0000275.0000285.0000295.0000
305.0000315.0000325.0000335.0000345.0000355.0000
  5.0191   0.0000   3.8637   0.0000   2.3662   0.0000   1.7434   0.0000   1.4142   0.0000
  1.2208   0.0000   1.1034   0.0000   1.0353   0.0000   1.0038   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000
$STOP
/*
//
```

PRINTED OUTPUT

```
  N   M NK NG
 10 36  1  1

BK
 0.2908882E+00

X
  0.0000   5.3608  10.6048  15.5027  19.2648  19.2648  15.5027  10.6048   5.3608   0.0000

Y
-10.0000  -9.6320  -8.4706  -6.2894  -2.5559   2.5559   6.2894   8.4706   9.6320  10.0000

PHI
  5.0000  15.0000  25.0000  35.0000  45.0000  55.0000  65.0000  75.0000  85.0000  95.0000
105.0000115.0000125.0000135.0000145.0000155.0000165.0000175.0000185.0000195.0000
205.0000215.0000225.0000235.0000245.0000255.0000265.0000275.0000285.0000295.0000
305.0000315.0000325.0000335.0000345.0000355.0000
```

```
G
  5.0191   0.0000   3.8637   0.0000   2.3662   0.0000   1.7434   0.0000   1.4142   0.0000
  1.2208   0.0000   1.1034   0.0000   1.0353   0.0000   1.0038   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000

GG= 0.5554509E+02 FF= 0.1336629E+02 E= 0.3121504E+00 0= 0.1259270E+02

F
-0.5421209E 00 0.5224890E-01 0.1187810E 01 0.5969831E 00-0.1467048E 01
-0.9448900E 00 0.6552452E 00 0.1432544E 01-0.3653175E 00-0.7956181E 00
-0.9229600E-02 0.1055890E 01-0.3044195E 00-0.8306894E 00 0.8837318E-01
 0.1247218E 01-0.7381020E 00-0.8539429E 00 0.5231936E 00 0.2333546E-01

FM
  0.312   0.762   1.000   0.903   0.502   0.605   0.507   0.717   0.647   0.300

FP
  -38.3  173.9     0.0 -147.4    32.6 -122.3    37.1 -126.8    16.4  149.8

TF
  2.5665   0.1147   3.1540  -0.0913   2.7786  -0.4051   1.7283  -0.4121   0.7716   0.0494
  0.4798   0.5563   0.6498   0.5719   0.6821   0.1756   0.4518  -0.1075   0.3169  -0.1359
  0.3586  -0.2250   0.2700  -0.4598  -0.0353  -0.5732  -0.2900  -0.4708  -0.3163  -0.3647
 -0.2018  -0.4032  -0.0808  -0.4898  -0.0044  -0.4700  -0.0049  -0.3379  -0.1089  -0.2402
 -0.2452  -0.3117  -0.2631  -0.5031  -0.1208  -0.6144   0.0478  -0.5477   0.1494  -0.4074
  0.2410  -0.2631   0.2951  -0.0465   0.1469   0.1761  -0.1679   0.1827  -0.4018  -0.0090
 -0.5147  -0.1244  -0.6892  -0.0737  -0.8827  -0.0299  -0.7285  -0.0679   0.0563  -0.0556
  1.3298   0.0654

TFM
  2.5691   3.1553   2.8080   1.7767   0.7732   0.7346   0.8656   0.7044   0.4644   0.3449
  0.4233   0.5333   0.5743   0.5530   0.4828   0.4509   0.4964   0.4700   0.3380   0.2637
  0.3966   0.5677   0.6262   0.5498   0.4339   0.3568   0.2987   0.2294   0.2481   0.4019
  0.5295   0.6931   0.8832   0.7317   0.0792   1.3314
```

III. MAGNITUDE PATTERN SYNTHESIS

The program of this section implements the field magnitude pattern synthesis described in section IV of Report No. 2.

Punched card data is read in according to

```
      READ(1,30) N,M,NK,NG,N9
30    FORMAT(5I3)
      READ(1,33)(BK(I), I=1, NK)
33    FORMAT(5E14.7)
      READ(1,10)(X(I), I=1, N)
10    FORMAT(10F8.4)
      READ(1,10)(Y(I), I=1, N)
      READ(1,10)(PHI(I), I=1, M)
      DO 35 JK=1, NK
      DO 36 JG=1, NG
      READ(1,10)(G(I), I=1, M)
36    CONTINUE
35    CONTINUE
```

The x and y coordinates of N point sources are read in as X and Y. The starting value of $h_m e^{j\beta_m}$ in (2-21) is read in as G at M angles PHI in degrees in the plane of the point sources. The magnitude of G(m) is the magnitude of the specified pattern at the m^{th} angle. The angular portion $e^{j\beta_m}$ of G(m) gives the starting value for β_m. The index JK of DO loop 35 denotes the JK^{th} propagation constant BK(JK). DO loop 36 makes it possible to deal successively with NG different specified patterns at the same propagation constant BK(JK). Step 2 of the iteration procedure described in section IV, Report No. 2, is repeatedly executed until the pattern synthesis error ε of (2-19) ceases to decrease, but not more than a maximum of N9 times.

Minimum allocations are given by

```
COMPLEX C(N*N)
DIMENSION LR(N)
```

in the subroutine LINEQ and by

```
COMPLEX G(M), T(N*M), TC(N*M), TT(N*N),
    F(N), TF(M)
DIMENSION BK(NK), X(N), Y(N), PHI(M), TFM(M),
    H(M), E(N9+1), Q(N9+1), FF(N9+1),
    FM(N), FP(N)
```

in the main program.

Nested DO loops 14 and 15 store the matrix element given by (2-18) in T((m-1)*N+n) and its complex conjugate in TC(m-1)*N+n). Assuming that [W] is the identity matrix, nested DO loops 16 and 17 store the matrix $[\tilde{T}^*T]$ of (2-16) by columns in TT. The subroutine LINEQ inverts the N by N matrix stored in TT. DO loop 19 stores $[\tilde{T}^*T]^{-1}\tilde{T}^*$ of (2-16) by columns in TC.

DO loop 40 stores $\vec{h}$ of (2-19) in H and accumulates $||\vec{h}||^2$ in GG. Steps 1, 2, and 3 of the iteration procedure described in section IV, Report No. 2, are executed for the J9th time inside DO loop 41. DO loop 42 stores $\vec{f}$ of (2-16) in F and accumulates $||\vec{f}||^2$ in FF(J2). DO loop 51 stores $[T]\vec{f}$ of (2-19) in TF, $|[T]\vec{f}|$ in TFM, $||[T]\vec{f}||^2$ in TFS, and the pattern synthesis error ε of (2-19) in S1. DO loop 51 also changes the phase of G to that of $[T]\vec{f}_i$. Since β_m appears only in $e^{j\beta_m}$, the iteration procedure described in section IV, Report No. 2,can be carried out without calculating β_m explicitly. Just after DO loop 51, Q of (2-15) is stored in Q(J2) and the relative pattern synthesis error $\frac{\varepsilon}{||\vec{h}||^2}$ in E(J2). Execution remains in DO loop 41 only as long as the relative error continues to decrease.

Statement 59 punches F on cards for possible input into the program of section IX. DO loop 55 finds that F(J2) is largest in magnitude. DO loop 56 normalizes the element of F largest in magnitude to unity and then stores the magnitudes and phases in degrees of F in FM and FP respectively.

The magnitude of the last element of $\vec{f}$ is 0.235 in the sample output of the magnitude pattern synthesis program as opposed to 0.234 in the table of section IV, Report No. 2. This slight difference is due to the fact that the data for the table of section IV, Report No. 2, was computed with FORTG instead of WATFIV. The synthesized patterns were also slightly different. For instance, the eleventh element of TFM was 0.0811 with FORTG as opposed to 0.0816 with WATFIV.

LISTING OF MAGNITUDE PATTERN SYNTHESIS PROGRAM

```
//               (0034,EE,20S,1,4),'MAUTZ,JOE',REGION=200K
// EXEC WATFIV
//GO.FT02F001 DD SYSOUT=B,DCB=(RECFM=F,BLKSIZE=80)
//GO.SYSIN DD *
$JOB             MAUTZ,TIME=1,PAGES=40
      SUBROUTINE LINEQ(LL,C)
      COMPLEX C(100),STOR,STO,ST,S
      DIMENSION LR(50)
      DO 20 I=1,LL
      LR(I)=I
   20 CONTINUE
      M1=0
      DO 18 M=1,LL
      K=M
      DO 2 I=M,LL
      K1=M1+I
      K2=M1+K
      IF(ABS(REAL(C(K1)))+ABS(AIMAG(C(K1)))-ABS(REAL(C(K2)))-ABS(AIMAG(C
     1(K2)))) 2,2,6
    6 K=I
    2 CONTINUE
      LS=LR(M)
      LR(M)=LR(K)
      LR(K)=LS
      K2=M1+K
      STOR=1./C(K2)
      J1=0
      DO 7 J=1,LL
      K1=J1+K
      K2=J1+M
      STO=C(K1)
      C(K1)=C(K2)
      C(K2)=STO*STOR
      J1=J1+LL
    7 CONTINUE
      K1=M1+M
      C(K1)=STOR
      DO 11 I=1,LL
      IF(I-M) 12,11,12
   12 K1=M1+I
      ST=C(K1)
      C(K1)=0.
      J1=0
      DO 10 J=1,LL
      K1=J1+I
      K2=J1+M
      C(K1)=C(K1)-C(K2)*ST
      J1=J1+LL
   10 CONTINUE
   11 CONTINUE
      M1=M1+LL
   18 CONTINUE
      J1=0
      DO 9 J=1,LL
      IF(J-LR(J)) 14,8,14
   14 LRJ=LR(J)
      J2=(LRJ-1)*LL
   21 DO 13 I=1,LL
```

```
      K2=J2+I
      K1=J1+I
      S=C(K2)
      C(K2)=C(K1)
      C(K1)=S
   13 CONTINUE
      LR(J)=LR(LRJ)
      LR(LRJ)=LRJ
      IF(J-LR(J)) 14,8,14
    8 J1=J1+LL
    9 CONTINUE
      RETURN
      END
      COMPLEX U,U1,CONJG,G(36),T(360),TC(360),TT(100),F(10),TF(36)
      DIMENSION BK(10),X(10),Y(10),PHI(36),TFM(36),H(36),E(61),O(61)
      DIMENSION FF(61),FM(10),FP(10)
      U=(0.,1.)
      PI=3.141593
      C=PI/180.
      READ(1,30) N,M,NK,NG,N9
   30 FORMAT(5I3)
      WRITE(3,32) N,M,NK,NG,N9
   32 FORMAT('0   N   M  NK  NG  N9'/1X,5I3)
      READ(1,33)(BK(I),I=1,NK)
   33 FORMAT(5E14.7)
      WRITE(3,34)(BK(I),I=1,NK)
   34 FORMAT('0BK'/(1X,5E14.7))
      READ(1,10)(X(I),I=1,N)
   10 FORMAT(10F8.4)
      WRITE(3,13)(X(I),I=1,N)
   13 FORMAT('0X'/(1X,10F8.4))
      READ(1,10)(Y(I),I=1,N)
      WRITE(3,37)(Y(I),I=1,N)
   37 FORMAT('0Y'/(1X,10F8.4))
      READ(1,10)(PHI(I),I=1,M)
      WRITE(3,11)(PHI(I),I=1,M)
   11 FORMAT('0PHI'/(1X,10F8.4))
      ZM=M
      DO 35 JK=1,NK
      BB=BK(JK)
      J1=0
      DO 14 K=1,M
      S1=PHI(K)*C
      CS=COS(S1)
      SN=SIN(S1)
      DO 15 I=1,N
      J1=J1+1
      S1=BB*(X(I)*CS+Y(I)*SN)
      T(J1)=COS(S1)+U*SIN(S1)
      TC(J1)=CONJG(T(J1))
   15 CONTINUE
   14 CONTINUE
      J1=0
      DO 16 J=1,N
      DO 17 I=1,N
      J2=J
      J3=I
      J1=J1+1
      TT(J1)=0.
      DO 18 K=1,M
```

```
      TT(J1)=TT(J1)+TC(J3)*T(J2)
      J3=J3+N
      J2=J2+N
   18 CONTINUE
   17 CONTINUE
   16 CONTINUE
      CALL LINEQ(N,TT)
      J1=0
      J3=0
      DO 19 J=1,M
      DO 39 I=1,N
      J3=J3+1
      F(I)=TC(J3)
   39 CONTINUE
      DO 20 I=1,N
      J1=J1+1
      TC(J1)=0.
      J2=I
      DO 38 K=1,N
      TC(J1)=TC(J1)+TT(J2)*F(K)
      J2=J2+N
   38 CONTINUE
   20 CONTINUE
   19 CONTINUE
      DO 36 JG=1,NG
      READ(1,10)(G(I),I=1,M)
      WRITE(3,12)(G(I),I=1,M)
   12 FORMAT('0G'/(1X,10F8.4))
      GG=0.
      DO 40 J=1,M
      S1=G(J)*CONJG(G(J))
      H(J)=SQRT(S1)
      GG=GG+S1
   40 CONTINUE
      WRITE(3,47) GG
   47 FORMAT('0NORM SQUARED OF SPECIFIED PATTERN=',E14.7)
      FF(1)=0.
      F(1)=1.
      Q(1)=0.
      DO 41 J9=1,N9
      J2=J9+1
      FF(J2)=0.
      DO 42 J=1,N
      J1=J
      F(J)=0.
      DO 43 I=1,M
      F(J)=F(J)+TC(J1)*G(I)
      J1=J1+N
   43 CONTINUE
      FF(J2)=FF(J2)+F(J)*CONJG(F(J))
   42 CONTINUE
      J1=0
      TFS=0.
      S1=0.
      DO 51 J=1,M
      TF(J)=0.
      DO 45 I=1,N
      J1=J1+1
      TF(J)=TF(J)+T(J1)*F(I)
   45 CONTINUE
```

```
      S2=TF(J)*CONJG(TF(J))
      TFM(J)=SQRT(S2)
      IF(TFM(J).EQ.0.) GO TO 52
      TFS=TFS+S2
      G(J)=(H(J)/TFM(J))*TF(J)
   52 S2=TFM(J)-H(J)
      S1=S1+S2*S2
   51 CONTINUE
      Q(J2)=ZM*FF(J2)/TFS
      E(J2)=S1/GG
      IF(E(J2).GE.E(J9)) GO TO 44
   41 CONTINUE
   44 WRITE(3,54) FF(I),I=1,J2)
   54 FORMAT('ONORM SQUARED OF F'/(1X,5E14.7))
      WRITE(3,46)(E(I),I=1,J2)
   46 FORMAT('ORELATIVE ERROR'/(1X,5E14.7))
      WRITE(3,53)(Q(I),I=1,J2)
   53 FORMAT('0Q'/(1X,5F14.7))
      WRITE(3,48)(F(I),I=1,N)
   48 FORMAT('0F'/(1X,5F14.7))
   59 WRITE(2,33)(F(I),I=1,N)
      S2=0.
      DO 55 J=1,N
      S1=F(J)*CONJG(F(J))
      IF(S1.LT.S2) GO TO 55
      S2=S1
      J2=J
   55 CONTINUE
      U1=1./F(J2)
      DO 56 J=1,N
      F(J)=U1*F(J)
      FM(J)=CABS(F(J))
      S1=REAL(F(J))
      S2=AIMAG(F(J))
      FP(J)=(ATAN2(S2,S1))/C
   56 CONTINUE
      WRITE(3,57)(FM(J),J=1,N)
   57 FORMAT('0FM'/(1X,10F7.3))
      WRITE(3,58)(FP(J),J=1,N)
   58 FORMAT('0FP'/(1X,10F7.1))
      WRITE(3,49)(TF(J),J=1,M)
   49 FORMAT('0TF'/(1X,10F8.4))
      WRITE(3,50)(TFM(J),J=1,M)
   50 FORMAT('0TFM'/(1X,10F8.4))
   36 CONTINUE
   35 CONTINUE
      STOP
      END
$DATA
 10 36  1  1 40
 0.2908882E+00
  0.0000   5.3608 10.6048 15.5027 19.2648 19.2648 15.5027 10.6048  5.3608  0.0000
-10.0000 -9.6320 -8.4706 -6.2894 -2.5559  2.5559  6.2894  8.4706  9.6320 10.0000
  5.0000 15.0000 25.0000 35.0000 45.0000 55.0000 65.0000 75.0000 85.0000 95.0000
105.0000115.0000125.0000135.0000145.0000155.0000165.0000175.0000185.0000195.0000
205.0000215.0000225.0000235.0000245.0000255.0000265.0000275.0000285.0000295.0000
305.0000315.0000325.0000335.0000345.0000355.0000
  5.0191   0.0000   3.8637   0.0000   2.3662   0.0000   1.7434   0.0000   1.4142   0.0000
  1.2208   0.0000   1.1034   0.0000   1.0353   0.0000   1.0038   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
```

```
  0.0000   0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000   0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000   0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000   0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000   0.0000
$STOP
/*
//

PRINTED OUTPUT
  N  M NK NG N9
 10 36  1  1 40

BK
 0.2908882E+00

X
  0.0000  5.3608 10.6048 15.5027 19.2648 19.2648 15.5027 10.6048  5.3608  0.0000

Y
-10.0000 -9.6320 -8.4706 -6.2894 -2.5559  2.5559  6.2894  8.4706  9.6320 10.0000

PHI
  5.0000 15.0000 25.0000 35.0000 45.0000 55.0000 65.0000 75.0000 85.0000 95.0000
105.0000115.0000125.0000135.0000145.0000155.0000165.0000175.0000185.0000195.0000
205.0000215.0000225.0000235.0000245.0000255.0000265.0000275.0000285.0000295.0000
305.0000315.0000325.0000335.0000345.0000355.0000

G
  5.0191   0.0000  3.8637  0.0000  2.3662  0.0000  1.7434  0.0000  1.4142  0.0000
  1.2208   0.0000  1.1034  0.0000  1.0353  0.0000  1.0038  0.0000  0.0000  0.0000
  0.0000   0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000   0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000   0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000   0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000   0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000   0.0000

NORM SQUARED OF SPECIFIED PATTERN= 0.5554509E+02

NORM SQUARED OF F
 0.0000000E+00 0.1336623E+02 0.1819649E+02 0.2036316E+02 0.2223022E+02
 0.2658525E+02 0.3068034E+02 0.3176707E+02 0.3115558E+02 0.2976671E+02
 0.2860146E+02 0.2795184E+02 0.2765752E+02 0.2754288E+02 0.2750768E+02
 0.2750438E+02 0.2751225E+02 0.2752257E+02 0.2753236E+02 0.2754047E+02

RELATIVE ERROR
 0.1000000E+01 0.2848189E+00 0.2568424E+00 0.2447110E+00 0.2361352E+00
 0.2238368E+00 0.2103522E+00 0.1953782E+00 0.1817446E+00 0.1748158E+00
 0.1727168E+00 0.1722322E+00 0.1721287E+00 0.1721053E+00 0.1720990E+00
 0.1720966E+00 0.1720955E+00 0.1720948E+00 0.1720945E+00 0.1720946E+00

Q
 0.0000000E+00 0.1259257E+02 0.1609077E+02 0.1758795E+02 0.1896851E+02
 0.2237906E+02 0.2538416E+02 0.2582610E+02 0.2485503E+02 0.2344670E+02
 0.2242101E+02 0.2188480E+02 0.2164862E+02 0.2155760E+02 0.2152974E+02
 0.2152713E+02 0.2153319E+02 0.2154135E+02 0.2154883E+02 0.2155528E+02

F
-0.6131346E+00-0.2752376E+00 0.1244744E+01 0.9962800E+00-0.1532891E+01
```

```
-0.1988470E+01 0.4787318E+00 0.2502643E+01 0.2811250E+00-0.1595194E+01
-0.3373244E+00 0.1369034E+01 0.2236381E+00-0.1435582E+01 0.1607297E-01
 0.1691813E+01-0.4139780E+00-0.1279405E+01 0.4812400E+00 0.3546119E+00

FM
  0.264   0.626   0.985   1.000   0.636   0.553   0.570   0.664   0.528   0.235

FP
  125.0   -40.5   153.2     0.0  -159.2    24.7  -160.3    10.3   172.9   -42.8

TF
  3.0116 -0.0509  3.4498 -0.6797  2.5152 -1.4827  0.5756 -1.6077 -1.1052 -0.5666
 -1.3524  0.9370 -0.3466  1.5845  0.5760  1.0169  0.6059  0.1924  0.2263 -0.0625
  0.0778  0.0247  0.0547 -0.0063 -0.1166 -0.0908 -0.2980 -0.0544 -0.2767 -0.0220
 -0.1164 -0.1273  0.0100 -0.2474  0.0312 -0.1986 -0.0444 -0.0116 -0.1644  0.0965
 -0.2082 -0.0035 -0.0926 -0.1687  0.0510 -0.1782  0.0181 -0.0895 -0.0958 -0.1372
 -0.0114 -0.2522  0.1820 -0.1469  0.1350  0.0756 -0.0859  0.0453 -0.1245 -0.1601
 -0.0508 -0.1402 -0.2286  0.1054 -0.5787  0.2217 -0.5413  0.1203  0.2762  0.0504
  1.6839  0.0364

TFM
  3.0120  3.5161  2.9197  1.7076  1.2420  1.6453  1.6220  1.1687  0.6357  0.2347
  0.0816  0.0551  0.1478  0.3029  0.2775  0.1725  0.2476  0.2011  0.0459  0.1906
  0.2082  0.1325  0.1854  0.0913  0.1674  0.2525  0.2339  0.1547  0.0971  0.2026
  0.1491  0.2517  0.6197  0.5545  0.2808  1.6867
```

IV. EIGENVALUES AND EIGENVECTORS OF $[\tilde{T}^*T]$

The program of this section punches on cards the eigenvalues λ_i and the eigenvectors $\vec{\phi}_i$ of (2-35) when W and V are identity matrices.

Punched card data is read in according to

```
      READ(1,30) N,M,BK
30    FORMAT(2I3, E14.7)
      READ(1,10)(X(I), I=1, N)
10    FORMAT(10F8.4)
      READ(1,10)(Y(I), I=1,N)
      READ(1,10)(PHI(I), I=1, M)
```

The x and y coordinates of N point sources are read in as X and Y. The M angles ϕ_m of (2-18) are read in as PHI. The propagation constant is BK.

Minimum allocations are given by

```
COMPLEX A(N*(N+1)/2), R(N*N)
```

in the subroutine CEIG and by

```
COMPLEX T(N*M), TC(N*M), A(N*(N+1)/2),
   TT(N*N), P(N*N)
DIMENSION X(N), Y(N), PHI(M), AM(N)
```

in the main program.

Nested DO loops 14 and 15 store the matrix element given by (2-18) in T((m-1)*N+n) and its complex conjugate in TC((m-1)*N+n). Assuming that [W] is the identity matrix, nested DO loops 16 and 17 store the upper triangular portion of $[\tilde{T}^*T]$ of (2-35) in A and the whole matrix $[\tilde{T}^*T]$ by columns in TT.

The subroutine CEIG called by statement 40 puts the eigenvalues of $[\tilde{T}^*T]$ in decreasing order in the diagonal positions of A and stores the i^{th} component of the j^{th} eigenvector of $[\tilde{T}^*T]$ in P((j-1)*N+i). The eigenvectors calculated by CEIG satisfy the orthogonality relationships (2-36) and (2-37). If the fourth argument MV of CEIG is not 1, the eigenvectors

are stored in the second argument R. If MV=1, no eigenvectors are stored. CEIG is an extension to Hermitian matrices of the subroutine EIGEN in the IBM Scientific Subroutine Package, [2], [3]. A detailed description of CEIG is not included in this report.

DO loop 38 stores the eigenvalues in AM. Statements 41 and 42 punch the eigenvalues and eigenvectors on cards. To check the eigenvalues, DO loop 28 stores in AM(J) the quadratic form $\tilde{\phi}_J^*[\tilde{T}^*T]\vec{\phi}_J$ which would be the Rayleigh quotient for λ_J if $\check{\phi}_J^* \vec{\phi}_J = 1$.

According to the discussion in section VII, Report No. 2, the eigenvectors in the sample printed output are supposed to be purely real. However, because of round off error, the computed elements of $[\tilde{T}^*T]$ and thus the eigenvectors have small imaginary parts.

---

[2] IBM System/360 Scientific Subroutine Package (360A-CM-03X) Version III, Programmer's manual.

[3] C. Fröberg, "Introduction to Numerical Analysis, Addison-Wesley, 1965, pp. 109-112.

LISTING OF PROGRAM TO PUNCH EIGENVALUES AND EIGENVECTORS

```
//                (0034,FE,15S,1,42),'MAUTZ,JOE',REGION=200K
// EXEC WATFIV
//GO.FT02F001 DD SYSOUT=B,DCB=(RECFM=F,BLKSIZE=80)
//GO.SYSIN DD *
$JOB              MAUTZ,TIME=1,PAGES=40
      SUBROUTINE CEIG(A,R,N,MV)
      COMPLEX A(55),R(100)
      COMPLEX U,U1,U2,U3,CONJG
      U=(0.,1.)
      MV1=MV-1
      IF(MV1) 10,25,10
   10 IJ=0
      DO 20 J=1,N
      J1=IJ+J
      DO 21 I=1,N
      IJ=IJ+1
      R(IJ)=0.
   21 CONTINUE
      R(J1)=1.
   20 CONTINUE
   25 ANORM=0.
      IA=1
      DO 35 J=2,N
      J1=J-1
      DO 36 I=1,J1
      IA=IA+1
      ANORM=ANORM+A(IA)*CONJG(A(IA))
   36 CONTINUE
      IA=IA+1
   35 CONTINUE
      FN=N
      IF(ANORM) 165,165,40
   40 ANORM=1.414*SQRT(ANORM)
      ANRMX=ANORM*1.E-6/FN
      IND=0
      THR=ANORM
   45 THR=THR/FN
  127 LM=0
   51 IMQ=0
      DO 55 M=2,N
      IMQ=IMQ+N
      J1=M-1
      LM=LM+1
      MQ=LM
      MM=MQ+M
      LL=0
      ILQ=-N
      DO 50 L=1,J1
      ILQ=ILQ+N
      LM=LM+1
      LQ=LL
      LL=LL+L
      RA=REAL(A(LM))
      AA=AIMAG(A(LM))
      AM2=RA*RA+AA*AA
      AM=SQRT(AM2)
      IF(AM-THR) 50,65,65
   65 IND=1
```

```
      ALL=REAL(A(LL))
      AMM=REAL(A(MM))
      X=.5*(ALL-AMM)
      SQ=SQRT(AM2+X*X)
      SINX=-SQRT(AM2/(2.*SQ*(SQ+ABS(X))))
      IF(X) 70,75,75
   70 SINX=-SINX
   75 SINX2=SINX*SINX
      COSX2=1.-SINX2
      COSX=SQRT(COSX2)
      SINCS=SINX*COSX
      SINT=AA/AM
      COST=RA/AM
      U1=SINX*(COST+U*SINT)
      U2=CONJG(U1)
      IQ=0
      DO 126 I=1,N
      IMM=I-M
      IF(I-L) 80,115,81
   80 IF(IMM) 82,115,83
   82 IM=I+MQ
      IL=I+LQ
      U3=A(IL)*COSX-A(IM)*U2
      A(IM)=A(IL)*U1+A(IM)*COSX
      GO TO 124
   83 IM=M+IQ
      IL=I+LQ
      U3=A(IL)*COSX-CONJG(A(IM))*U2
      A(IM)=CONJG(A(IL))*U2+A(IM)*COSX
      GO TO 124
   81 IF(IMM) 84,115,85
   84 IM=I+MQ
      IL=L+IQ
      U3=A(IL)*COSX-CONJG(A(IM))*U1
      A(IM)=CONJG(A(IL))*U1+A(IM)*COSX
      GO TO 124
   85 IM=M+IQ
      IL=L+IQ
      U3=A(IL)*COSX-A(IM)*U1
      A(IM)=A(IL)*U2+A(IM)*COSX
  124 A(IL)=U3
  115 IF(MV1) 120,125,120
  120 ILR=ILQ+I
      IMR=IMQ+I
      U3=R(ILR)*COSX-R(IMR)*U2
      R(IMR)=R(ILR)*U1+R(IMR)*COSX
      R(ILR)=U3
  125 IQ=IQ+I
  126 CONTINUE
      X=2.*AM*SINCS
      Y=ALL*COSX2+AMM*SINX2-X
      X=ALL*SINX2+AMM*COSX2+X
      A(LM)=A(LM)*COSX2+U1*((ALL-AMM)*COSX-CONJG(A(LM))*U1)
      A(LL)=Y
      A(MM)=X
   50 CONTINUE
   55 CONTINUE
      IF(IND-1) 160,155,160
  155 IND=0
      GO TO 127
```

```
160 IF(THR-ANRMX) 165,165,45
165 IQ=-N
    LL=0
    DO 185 I=1,N
    JQ=IQ
    IQ=IQ+N
    MM=LL
    LL=LL+I
    DO 185 J=I,N
    JQ=JQ+N
    MM=MM+J
    ALL=REAL(A(LL))
    AMM=REAL(A(MM))
    IF(ALL-AMM) 170,185,185
170 A(LL)=AMM
    A(MM)=ALL
    IF(MV1) 175,185,175
175 DO 180 K=1,N
    ILR=IQ+K
    IMR=JQ+K
    U3=R(ILR)
    R(ILR)=R(IMR)
180 R(IMR)=U3
185 CONTINUE
131 RETURN
    END
    COMPLEX U,U1,CONJG,T(360),TC(360),A(55),TT(100),P(100)
    DIMENSION X(10),Y(10),PHI(36),AM(10)
    PI=3.141593
    C=PI/180.
    U=(0.,1.)
    READ(1,30) N,M,BK
 30 FORMAT(2I3,E14.7)
    WRITE(3,32) N,M,BK
 32 FORMAT('0  N  M',6X,'BK'/1X,2I3,E14.7)
    READ(1,10)(X(I),I=1,N)
 10 FORMAT(10F8.4)
    WRITE(3,13)(X(I),I=1,N)
 13 FORMAT('0X'/(1X,10F8.4))
    READ(1,10)(Y(I),I=1,N)
    WRITE(3,37)(Y(I),I=1,N)
 37 FORMAT('0Y'/(1X,10F8.4))
    READ(1,10)(PHI(I),I=1,M)
    WRITE(3,11)(PHI(I),I=1,M)
 11 FORMAT('0PHI'/(1X,10F8.4))
    J1=0
    DO 14 K=1,M
    S1=PHI(K)*C
    CS=COS(S1)
    SN=SIN(S1)
    DO 15 I=1,N
    J1=J1+1
    S1=BK*(X(I)*CS+Y(I)*SN)
    T(J1)=COS(S1)+U*SIN(S1)
    TC(J1)=CONJG(T(J1))
 15 CONTINUE
 14 CONTINUE
    J1=0
    J5=0
    DO 16 J=1,N
```

```
      J4=J
      DO 17 I=1,J
      J2=J
      J3=I
      J1=J1+1
      A(J1)=0.
      DO 18 K=1,M
      A(J1)=A(J1)+TC(J3)*T(J2)
      J3=J3+N
      J2=J2+N
   18 CONTINUE
      J6=J5+I
      TT(J6)=A(J1)
      TT(J4)=CONJG(A(J1))
      J4=J4+N
   17 CONTINUE
      J5=J5+N
   16 CONTINUE
   40 CALL CEIG(A,P,N,0)
      J1=0
      DO 38 J=1,N
      J1=J1+J
      AM(J)=A(J1)
   38 CONTINUE
      WRITE(3,34)(AM(J),J=1,N)
   34 FORMAT('0EIGENVALUES'/(1X,5E14.7))
   41 WRITE(2,36)(AM(J),J=1,N)
   36 FORMAT(5E14.7)
      NN=N*N
      WRITE(3,35)(P(I),I=1,NN)
   35 FORMAT('0EIGENVECTORS'/(1X,5E14.7))
   42 WRITE(2,36)(P(I),I=1,NN)
      N1=N*(N+1)/2
      WRITE(3,26)(A(J),J=1,N1)
   26 FORMAT('0DIAGONAL MATRIX'/(1X,7E11.4))
      J1=0
      DO 28 J=1,N
      AM(J)=0.
      DO 29 I=1,N
      U1=0.
      J2=I
      DO 33 K=1,N
      J3=J1+K
      U1=U1+TT(J2)*P(J3)
      J2=J2+N
   33 CONTINUE
      J4=J1+I
      S1=U1*CONJG(P(J4))
      AM(J)=AM(J)+S1
   29 CONTINUE
      J1=J1+N
   28 CONTINUE
      WRITE(3,39)(AM(J),J=1,N)
   39 FORMAT('0RAYLEIGH QUOTIENTS'/(1X,5E14.7))
      STOP
      END
$DATA
 10 36 0.2908F82E+00
  0.0000  5.3608 10.6048 15.5027 19.2648 19.2648 15.5027 10.6048  5.3608  0.0000
-10.0000 -9.6320 -8.4706 -6.2894 -2.5559  2.5559  6.2894  8.4706  9.6320 10.0000
```

```
  5.0000 15.0000 25.0000 35.0000 45.0000 55.0000 65.0000 75.0000 85.0000 95.0000
105.0000115.0000125.0000135.0000145.0000155.0000165.0000175.0000185.0000195.0000
205.0000215.0000225.0000235.0000245.0000255.0000265.0000275.0000285.0000295.0000
305.0000315.0000325.0000335.0000345.0000355.0000
$STOP
/*
//
```

PRINTED OUTPUT

```
  N  M       BK
 10 36 0.2908882E+00

X
  0.0000  5.3608 10.6048 15.5027 19.2648 19.2648 15.5027 10.6048  5.3608  0.0000

Y
-10.0000 -9.6320 -8.4706 -6.2894 -2.5559  2.5559  6.2894  8.4706  9.6320 10.0000

PHI
  5.0000 15.0000 25.0000 35.0000 45.0000 55.0000 65.0000 75.0000 85.0000 95.0000
105.0000115.0000125.0000135.0000145.0000155.0000165.0000175.0000185.0000195.0000
205.0000215.0000225.0000235.0000245.0000255.0000265.0000275.0000285.0000295.0000
305.0000315.0000325.0000335.0000345.0000355.0000

EIGENVALUES
 0.9467145E+02 0.8078329E+02 0.6195313E+02 0.5468497E+02 0.2875992E+02
 0.2551213E+02 0.9639283E+01 0.3275412E+01 0.5507557E-00 0.1628423E+00

EIGENVECTORS
 0.2903522E 00 0.2304621E-06 0.6844866E-01 0.4241972E-08-0.3647421E 00
-0.9729786E-07-0.4885511E 00-0.2973702E-07-0.1982093E 00 0.4377566E-06
 0.1982076E 00 0.2547125E-06 0.4885510E 00-0.3091387E-06 0.3647433E 00
-0.1850908E-06-0.6844884E-01 0.2713567E-06-0.2903540E 00 0.5429575E-06
-0.8914965E-01 0.2161319E-06 0.2720940E 00-0.1533020E-06 0.3308787E 00
-0.9951651E-07-0.1172659E 00 0.3858715E-06-0.5429362E 00 0.2947568E-06
-0.5429378E 00-0.5368720E-07-0.1172673E 00-0.5244318E-06 0.3308780E 00
 0.3087853E-06 0.2720941E 00 0.7309694E-06-0.8914953E-01 0.2100297E-06
 0.5298515E 00 0.4270248E-06 0.4308088E 00 0.1473907E-05 0.2920523E-01
 0.9519529E-06-0.7784194E-01-0.5524618E-06 0.1635162E 00-0.8845636E-06
 0.1635150E 00 0.1771575E-06-0.7784206E-01 0.4654154E-06 0.2921355E-01
-0.6627855E-06 0.4308203E 00-0.1429648E-05 0.5298546E 00-0.7521597E-06
-0.1047320E 00-0.1435871E-05-0.5457036E 00-0.1159389E-05-0.3948668E 00
 0.1412610E-06 0.8893049E-01 0.4475079E-06 0.1655635E 00-0.7249168E-06
-0.1655665E 00-0.8152254E-06-0.8892918E-01 0.5292864E-06 0.3948660E 00
 0.6803169E-06 0.5456957E 00-0.5820197E-06 0.1047221E 00-0.1504132E-05
-0.3773477E-01-0.2794079E-05 0.1875506E-01-0.4097609E-06-0.4167684E 00
 0.3626927E-05-0.5573131E 00 0.2356654E-05-0.1180260E 00-0.1477333E-05
-0.1180170E 00 0.3259473E-05-0.5573068E 00 0.5574124E-05-0.4167722E 00
 0.2242483E-05 0.1875898E-01 0.2413907E-06-0.3772328E-01 0.3268949E-05
-0.5359712E 00-0.3648856E-05-0.1195107E 00-0.8504847E-06 0.1060919E 00
-0.1918221E-05-0.2810081E 00-0.5109343E-05-0.3289738E 00-0.2711899E-05
 0.3289769E 00 0.1486194E-05 0.2810201E 00-0.1367136E-05-0.1060811E 00
-0.3287122E-05 0.1195102E 00 0.6517278E-06 0.5359692E 00 0.3198825E-05
-0.4139768E 00-0.4251949E-06 0.3121096E 00 0.3563546E-06 0.2024003E 00
 0.1425478E-06-0.1954105E 00-0.4762701E-06 0.3899477E 00 0.6987133E-06
 0.3899392E 00-0.3403692E-06-0.1954079E 00 0.529444 E-07 0.2024022E 00
 0.2056303E-06 0.3121062E 00 0.1403968E-06-0.4139754E 00-0.2686328E-06
 0.3210983E 00 0.4192697E-06-0.3446192E 00-0.4561953E-06 0.2366264E 00
 0.2257797E-06 0.1475003E 00-0.4949951E-06-0.4476349E 00-0.4168046E-07
```

```
 0.4476404E 00-0.4944204E-06-0.1475034E 00-0.3288654E-06-0.2366209E 00
 0.2878805E-06 0.3446209E 00-0.5696532E-06-0.3211029E 00 0.5917625E-06
 0.1962254E 00 0.1733023E-05-0.3776212E 00-0.3023024E-05 0.4183334E 00
 0.3710524E-05-0.3625054E 00-0.3121309E-05 0.1117757E 00 0.1425577E-05
 0.1117630E 00-0.4275416E-06-0.3624925E 00-0.8278869E-07 0.4183226E 00
 0.1338545E-06-0.3776149E 00-0.3584536E-06 0.1962231E 00 0.1241534E-07
-0.1198015E 00-0.1655849E-06 0.2539174E 00 0.4111487E-06-0.3791949E 00
-0.3516973E-06 0.3907360E 00-0.3590495E-06-0.3531159E 00 0.8053731E-06
 0.3531210E 00-0.2384469E-05-0.3907486E 00 0.2968422E-05 0.3792086E 00
-0.3526511E-05-0.2539279E 00 0.2444993E-05 0.1198068E 00-0.1146047E-05
```

DIAGONAL MATRIX

```
 0.9467E+02 0.0000E+00 0.8562E-08 0.3283E-10 0.8078E+02 0.0000E+00-0.5781E-07
-0.2912E-11 0.2188E-05 0.3970E-10 0.6195E+02 0.0000E+00 0.5032E-05 0.1086E-08
-0.2737E-08 0.5581E-11-0.2503E-06 0.1073E-10 0.5468E+02 0.0000E+00-0.1738E-07
 0.1617E-11 0.3234E-06-0.4128E-10-0.8252E-05 0.5097E-09-0.8921E-06 0.4899E-09
 0.2876E+02 0.0000E+00-0.8160E-05-0.5499E-08-0.1506E-05 0.1549E-09 0.2023E-06
-0.2199E-11-0.3532E-07 0.6796E-11-0.1455E-10 0.0000E+00 0.2551E+02 0.0000E+00
 0.1455E-10-0.2487E-13 0.3169E-07-0.3171E-11 0.1544E-09 0.1287E-13 0.6015E-09
 0.1204E-11 0.5394E-05-0.4545E-09-0.7547E-09-0.6301E-14 0.9639E+01 0.0000E+00
-0.2062E-09-0.4100E-12-0.2913E-10 0.2229E-15-0.3578E-07-0.1377E-11-0.1360E-10
-0.5493E-13 0.2890E-05-0.8868E-10 0.2569E-05-0.1450E-09 0.0000E+00-0.1110E-14
 0.3275E+01 0.0000E+00 0.3713E-05 0.2476E-09-0.2506E-05-0.8876E-10-0.7184E-07
-0.1477E-10-0.2371E-09-0.1701E-12 0.3420E-09 0.6080E-13-0.6416E-06 0.3344E-09
 0.5390E-08-0.6532E-12-0.2782E-07 0.6371E-10 0.5508E+00 0.0000E+00-0.2881E-07
 0.2350E-11-0.1119E-07 0.1213E-11-0.1347E-10 0.3383E-14-0.4577E-05 0.2410E-09
-0.2328E-09 0.1554E-14 0.7932E-09 0.5311E-14-0.4533E-05 0.7766E-09-0.1236E-09
-0.3453E-12-0.1455E-10-0.6661E-15 0.1628E+00 0.0000E+00
```

RAYLEIGH QUOTIENTS

```
 0.9467105E+02 0.8078307E+02 0.6195314E+02 0.5468494E+02 0.2876013E+02
 0.2551244E+02 0.9639304E+01 0.3275400E+01 0.5507494E+00 0.1628391E+00
```

V. PATTERN SYNTHESIS WITH CONSTRAINED SOURCE NORM

The program of this section implements the theory of section V of Report No. 2. Punched card input data is read into the program according to

```
      READ(1,30) N,M, NG, BK, EPS
30    FORMAT (3I3, 2E14.7)
      READ(1,10)(X(I), I=1, N)
10    FORMAT(10F8.4)
      READ(1,10)(Y(I), I=1, N)
      READ(1,10)(PHI(I), I=1, M)
      READ(1,23)(AM(J), J=1, N)
23    FORMAT(5E14.7)
      NN = N*N
      READ(1,23)(P(J), J=1, NN)
      DO 22 JG=1, NG
      READ(1,23) CN
      READ(1,10)(G(I), I=1, M)
23    CONTINUE
```

The x and y coordinates of N point sources are read in as X and Y. The specified pattern $\vec{g}_o$ of (2-16) is read in as G at M angles PHI. The eigenvalues AM and eigenvectors P of the matrix $[\tilde{T}^*T]$ of (2-35) have been punched out by the program of section IV of this report. The constraint C of (2-24) is read in as CN. The iterative process for finding the root of (2-45) is terminated as soon as $F(\alpha) \leq$ EPS*CN. DO loop 22 makes it possible to deal successively with NG different specified patterns at the same propagation constant BK.

Minimum allocations are given by

```
COMPLEX P(N*N), T(N*M), G(M), TG(N), C(N),
   ALP(N), F(N), TF(M)
DIMENSION X(N), Y(N), PHI(M), AM(N), AM2(N),
   CC(N), TFM(M), FP(N), FM(N)
```

Nested DO loops 14 and 15 store the matrix element given by (2-18) in T((m-1)*N+n). DO loop 25 stores $\|\vec{g}_o\|^2$ in GG and replaces G by its complex conjugate. DO loop 20 stores the complex conjugate of $[\tilde{T}^*]\vec{g}_o$ of (2-42) in TG. Nested DO loops 26 and 27 store C_J of (2-42) in C(J) and $|C_J|^2$ in CC(J).

DO loop 31 stores F(0) of (2-45) in S6. If S6 < 0, DO loop 68 carries out Newton's method starting with α equal to the center eigenvalue. Referring to expression (2-45), α is stored in both AL and AS while F(α) is stored in FS. Statement 60 calculates

$$\alpha_{j+1} = \alpha_j - \frac{F(\alpha_j)}{F'(\alpha_j)} = \alpha_j - \frac{F(\alpha_j)}{2\sum_{i=1}^{N} \frac{|C_i|^2}{(\lambda_i+\alpha_j)^3}} \qquad (1)$$

If $\alpha_{j+1} \leq 0$ then α_{j+1} is set equal to zero. The subscripted α's appearing in (1) are not to be confused with those of (2-40). DO loop 91 puts α_i of (2-40) in ALP(i).

DO loop 33 stores $\vec{f}$ of (2-41) in F and accumulates $\|\vec{f}\|^2$ in FF. Upon exit from DO loop 33, F(J2) will be the element of $\vec{f}$ largest in magnitude. DO loop 92 stores the synthesized pattern $T\vec{f}$ of (2-23) in TF and the magnitudes of the elements $[T]\vec{f}$ in TFM. $\|\vec{f}\|^2$ is accumulated in TFS and ϵ of (2-23) in E. Just after DO loop 92, Q defined by (2-15) is stored in Q and E is normalized by dividing by GG. Statement 59 punches $\vec{f}$ on cards for possible input into the program of section IX of this report. DO loop 41 normalizes the element of $\vec{f}$ largest in magnitude to unity and then stores the magnitudes and phases in degrees of $\vec{f}$ in FM and FP respectively.

According to the printed outputs of the pattern synthesis with constrained source norm program of this section and the pattern synthesis program of section II of this report, it has been possible to reduce the source norm squared from 13.366 to 4.0 with only a very slight increase in the pattern synthesis error.

LISTING OF PATTERN SYNTHESIS WITH CONSTRAINED SOURCE NORM PROGRAM

```
//               (0034,EE,15S,1,4),'MAUTZ,JOE',REGION=200K
// EXEC WATFIV
//GO.FT02F001 DD SYSOUT=B,DCB=(RECFM=F,BLKSIZE=80)
//GO.SYSIN DD *
$JOB             MAUTZ,TIME=1,PAGES=40
      COMPLEX U,U1,CONJG,P(100),T(360),G(36),TG(10),C(10),ALP(10)
      COMPLEX U2,F(10),TF(36)
      DIMENSION X(10),Y(10),PHI(36),AM(10),AM2(10),CC(10),AS(31)
      DIMENSION FS(30),TFM(36),FP(10),FM(10)
      PI=3.141593
      CP=PI/180.
      U=(0.,1.)
      READ(1,30) N,M,NG,BK,EPS
   30 FORMAT(3I3,2E14.7)
      WRITE(3,32) N,M,NG,BK,EPS
   32 FORMAT('0  N  M NG',6X,'BK',11X,'EPS'/1X,3I3,2E14.7)
      READ(1,10)(X(I),I=1,N)
   10 FORMAT(10F8.4)
      WRITE(3,13)(X(I),I=1,N)
   13 FORMAT('0X'/(1X,10F8.4))
      READ(1,10)(Y(I),I=1,N)
      WRITE(3,37)(Y(I),I=1,N)
   37 FORMAT('0Y'/(1X,10F8.4))
      READ(1,10)(PHI(I),I=1,M)
      WRITE(3,11)(PHI(I),I=1,M)
   11 FORMAT('0PHI'/(1X,10F8.4))
      READ(1,23)(AM(J),J=1,N)
   23 FORMAT(5E14.7)
      WRITE(3,24)(AM(J),J=1,N)
   24 FORMAT('0AM'/(1X,5E14.7))
      NN=N*N
      ZM=M
      READ(1,23)(P(J),J=1,NN)
      WRITE(3,21)(P(J),J=1,NN)
   21 FORMAT('0P'/(1X,5E14.7))
      DO 29 J=1,N
      AM2(J)=AM(J)*AM(J)
   29 CONTINUE
      J1=0
      DO 14 K=1,M
      S1=PHI(K)*CP
      CS=COS(S1)
      SN=SIN(S1)
      DO 15 I=1,N
      J1=J1+1
      S1=BK*(X(I)*CS+Y(I)*SN)
      T(J1)=COS(S1)+U*SIN(S1)
   15 CONTINUE
   14 CONTINUE
      DO 22 JG=1,NG
      READ(1,23) CN
      WRITE(3,28) CN
   28 FORMAT('0CN=',E14.7)
      READ(1,10)(G(I),I=1,M)
      WRITE(3,12)(G(I),I=1,M)
   12 FORMAT('0G'/(1X,10F8.4))
      GG=0.
      DO 25 J=1,M
```

```
      U1=CONJG(G(J))
      GG=GG+G(J)*U1
      G(J)=U1
   25 CONTINUE
      DO 19 J=1,N
      J2=J
      TG(J)=0.
      DO 20 K=1,M
      TG(J)=TG(J)+T(J2)*G(K)
      J2=J2+N
   20 CONTINUE
   19 CONTINUE
      J1=0
      DO 26 J=1,N
      U1=0.
      DO 27 I=1,N
      J1=J1+1
      U1=U1+P(J1)*TG(I)
   27 CONTINUE
      C(J)=CONJG(U1)
      CC(J)=U1*C(J)
   26 CONTINUE
      WRITE(3,200)(CC(J),J=1,N)
  200 FORMAT('OCC'/(1X,5E14.7))
      JAM=(N+1)/2
      EP=EPS*CN
  201 S6=CN
      DO 31 K=1,N
      S6=S6-CC(K)/AM2(K)
   31 CONTINUE
      J9=1
      IF(S6) 66,65,65
   65 AL=0.
      AS(J9)=AL
      FS(J9)=S6
      GO TO 87
   66 AL=AM(JAM)
      AS(J9)=AL
      DO 68 J=1,30
      S2=CN
      S5=0.
      DO 67 K=1,N
      S3=AM(K)+AL
      S4=S3*S3
      S2=S2-CC(K)/S4
      S5=S5+CC(K)/(S4*S3)
   67 CONTINUE
      FS(J9)=S2
      IF(ABS(FS(J9)).LE.EP) GO TO 87
   60 AL=AS(J9)-.5*FS(J9)/S5
      IF(AL.LE.0.) AL=0.
      J9=J9+1
      AS(J9)=AL
   68 CONTINUE
      WRITE(3,219)
  219 FORMAT('ONEWTONS METHOD DID NOT CONVERGE')
   87 DO 91 K=1,N
      ALP(K)=C(K)/(AM(K)+AL)
   91 CONTINUE
      WRITE(3,90)(AS(K),K=1,J9)
```

```
 90 FORMAT(' CONVERGENCE OF LAGRANGE MULTIPLIER'/(1X,7E11.4))
    WRITE(3,100)(FS(K),K=1,J9)
100 FORMAT(' FUNCTION'/(1X,7E11.4))
    FF=0.
    S2=0.
    DO 33 I=1,N
    F(I)=0.
    J4=I
    DO 44 K=1,N
    F(I)=F(I)+ALP(K)*P(J4)
    J4=J4+N
 44 CONTINUE
    S1=F(I)*CONJG(F(I))
    FF=FF+S1
    IF(S1.LT.S2) GO TO 33
    S2=S1
    J2=I
 33 CONTINUE
    E=0.
    TFS=0.
    J1=0
    DO 92 KK=1,M
    U1=0.
    DO 93 K=1,N
    J1=J1+1
    U1=U1+T(J1)*F(K)
 93 CONTINUE
    TF(KK)=U1
    U2=CONJG(U1)
    S1=U1*U2
    TFS=TFS+S1
    TFM(KK)=SQRT(S1)
    U1=U2-G(KK)
    S1=U1*CONJG(U1)
    E=E+S1
 92 CONTINUE
    Q=ZM*FF/TFS
    E=E/GG
    WRITE(3,48) GG,FF,E,Q
 48 FORMAT('0GG=',E14.7,' FF=',E14.7,' E=',E14.7,' Q=',E14.7)
    WRITE(3,45)(F(I),I=1,N)
 45 FORMAT('OF'/(1X,5E14.7))
 59 WRITE(2,23)(F(I),I=1,N)
    U1=1./F(J2)
    DO 41 J=1,N
    F(J)=U1*F(J)
    FM(J)=CABS(F(J))
    S1=REAL(F(J))
    S2=AIMAG(F(J))
    FP(J)=(ATAN2(S2,S1))/CP
 41 CONTINUE
    WRITE(3,42)(FM(J),J=1,N)
 42 FORMAT('OFM'/(1X,10F7.3))
    WRITE(3,43)(FP(J),J=1,N)
 43 FORMAT('OFP'/(1X,10F7.1))
    WRITE(3,49)(TF(K),K=1,M)
 49 FORMAT('OTF'/(1X,10F8.4))
    WRITE(3,56)(TFM(K),K=1,M)
 56 FORMAT('OTFM'/(1X,10F8.4))
 22 CONTINUE
```

```
      STOP
      END
$DATA
 10 36   1 0.2904882E+00 0.1000000E-02
  0.0000   5.3608 10.6048 15.5027 19.2648 19.2648 15.5027 10.6048  5.3608  0.0000
-10.0000 -9.6320 -8.4706 -6.2894 -2.5559  2.5559  6.2894  8.4706  9.6320 10.0000
  5.0000 15.0000 25.0000 35.0000 45.0000 55.0000 65.0000 75.0000 85.0000 95.0000
105.0000115.0000125.0000135.0000145.0000155.0000165.0000175.0000185.0000195.0000
205.0000215.0000225.0000235.0000245.0000255.0000265.0000275.0000285.0000295.0000
305.0000315.0000325.0000335.0000345.0000355.0000
 0.9467145E 02 0.8078329E 02 0.6195313E 02 0.5468497E 02 0.2875992E 02
 0.2551213E 02 0.9639283E 01 0.3275412E 01 0.5507557E 00 0.1628423E 00
 0.2903522E 00 0.2304621E-06 0.6844866E-01 0.4241972E-08-0.3647421E 00
-0.9729786E-07-0.4885511E 00-0.2973702E-07-0.1982093E 00 0.4377566E-06
 0.1982076E 00 0.2547125E-06 0.4885510E 00-0.3091387E-06 0.3647433E 00
-0.1850908E-06-0.6844884E-01 0.2713567E-06-0.2903540E 00 0.5429575E-06
-0.8914965E-01 0.2161319E-06 0.2720940E 00-0.1533020E-06 0.3308787E 00
-0.9951651E-07-0.1172659E 00 0.3858715E-06-0.5429362E 00 0.2947568E-06
-0.5429378E 00-0.5368720E-07-0.1172673E 00-0.5244318E-06 0.3308780E 00
 0.3087853E-06 0.2720941E 00 0.7309694E-06-0.8914953E-01 0.2100297E-06
 0.5298515E 00 0.4270248E-06 0.4308088E 00 0.1473907E-05 0.2920523E-01
 0.9519529E-06-0.7784194E-01-0.5524618E-06 0.1635162E 00-0.8845636E-06
 0.1635150E 00 0.1771575E-06-0.7784205E-01 0.4654154E-06 0.2921355E-01
-0.6627855E-06 0.4308203E 00-0.1429648E-05 0.5298546E 00-0.7521597E-06
-0.1047320E 00-0.1435871E-05-0.5457036E 00-0.1159389E-05-0.3948668E 00
 0.1412510E-06 0.8893049E-01 0.4475079E-06 0.1655635E 00-0.7249168E-06
-0.1655655E 00-0.8152254E-06-0.8892918E-01 0.5292864E-06 0.3948660E 00
 0.6803169E-06 0.5456957E 00-0.5820197E-06 0.1047221E 00-0.1504132E-05
-0.3773477E-01-0.2794079E-05 0.1875506E-01-0.4097609E-06-0.4167684E 00
 0.3626927E-05-0.5573131E 00 0.2356654E-05-0.1180260E 00-0.1477333E-05
-0.1180170E 00 0.3259473E-05-0.5573068E 00 0.5574124E-05-0.4167722E 00
 0.2242483E-05 0.1875898E-01 0.2413907E-06-0.3772328E-01 0.3268949E-05
-0.5359712E 00-0.3648856E-05-0.1195107E 00-0.8504847E-06 0.1060919E 00
-0.1918221E-05-0.2810081E 00-0.5109343E-05-0.3289738E 00-0.2711899E-05
 0.3289769E 00 0.1486194E-05 0.2810201E 00-0.1367136E-05-0.1060811E 00
-0.3287122E-05 0.1195102E 00 0.6517278E-06 0.5359692E 00 0.3198825E-05
-0.4139768E 00-0.4251949E-06 0.3121096E 00 0.3563546E-06 0.2024003E 00
 0.1425478E-06-0.1954105E 00-0.4762701E-06 0.3899477E 00 0.6987133E-06
 0.3899392E 00-0.3403692E-06-0.1954079E 00 0.5294441E-07 0.2024022E 00
 0.2056303E-06 0.3121062E 00 0.1403968E-06-0.4139754E 00-0.2686328E-06
 0.3210983E 00 0.4192697E-06-0.3446192E 00-0.4561953E-06 0.2366264E 00
 0.2257797E-06 0.1475003E 00-0.4949951E-06-0.4476349E 00-0.4168046E-07
 0.4476404E 00-0.4944204E-06-0.1475034E 00-0.3288654E-06-0.2366209E 00
 0.2878805E-06 0.3446209E 00-0.5696532E-06-0.3211029E 00 0.5917625E-06
 0.1962254E 00 0.1733023E-05-0.3776212E 00-0.3023024E-05 0.4183334E 00
 0.3710524E-05-0.3625054E 00-0.3121309E-05 0.1117757E 00 0.1425577E-05
 0.1117630E 00-0.4275416E-06-0.3624925E 00-0.8278869E-07 0.4183226E 00
 0.1338545E-06-0.3776149E 00-0.3584536E-06 0.1962231E 00 0.1241534E-07
-0.1198015E 00-0.1655849E-06 0.2539174E 00 0.4111487E-06-0.3791949E 00
-0.3516973E-06 0.3907360E 00-0.3590495E-06-0.3531159E 00 0.8053731E-06
 0.3531210E 00-0.2384469E-05-0.3907486E 00 0.2968422E-05 0.3792086E 00
-0.3526511E-05-0.2539279E 00 0.2444993E-05 0.1198068E 00-0.1146047E-05
 0.4000000E+01
  5.0191   0.0000   3.8637   0.0000   2.3662   0.0000   1.7434   0.0000   1.4142   0.0000
  1.2208   0.0000   1.1034   0.0000   1.0353   0.0000   1.0038   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
```

```
 0.0000  0.0000
$STOP
/*
//

PRINTED OUTPUT
  N   M  NG        BK              EPS
 10  36   1 0.2908882E+00 0.1000000E-02

X
  0.0000   5.3608 10.6048 15.5027 19.2648 19.2648 15.5027 10.6048  5.3608  0.0000

Y
-10.0000 -9.6320 -8.4706 -6.2894 -2.5559  2.5559  6.2894  8.4706  9.6320 10.0000

PHI
  5.0000 15.0000 25.0000 35.0000 45.0000 55.0000 65.0000 75.0000 85.0000 95.0000
105.0000115.0000125.0000135.0000145.0000155.0000165.0000175.0000185.0000195.0000
205.0000215.0000225.0000235.0000245.0000255.0000265.0000275.0000285.0000295.0000
305.0000315.0000325.0000335.0000345.0000355.0000

AM
 0.9467145E 02 0.8078329E 02 0.6195313E 02 0.5468497E 02 0.2875992E 02
 0.2551213E 02 0.9639283E 01 0.3275412E 01 0.5507557E 00 0.1628423E 00

P
 0.2903522E+00 0.2304621E-06 0.6844866E-01 0.4241972E-08-0.3647421E+00
PLUS 39 MORE LINES OF PRINTED OUTPUT

CN= 0.4000000E+01

G
  5.0191   0.0000   3.8637   0.0000   2.3662   0.0000   1.7434   0.0000   1.4142   0.0000
  1.2208   0.0000   1.1034   0.0000   1.0353   0.0000   1.0038   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000

CC
 0.2552661E+03 0.4203154E+03 0.2834061E+02 0.2706128E+03 0.3613115E+03
 0.1311916E+03 0.1149906E+02 0.1260125E+02 0.2570739E+00 0.2755140E+00
CONVERGENCE OF LAGRANGE MULTPLIER
 0.2876E+02 0.0000E+00 0.7123E-01 0.1534E+00 0.2190E+00 0.2440E+00 0.2464E+00
FUNCTION
 0.3731E+01-0.9365E+01-0.3766E+01-0.1287E+01-0.2915E+00-0.2370E-01-0.1841E-03

GG= 0.5554509E+02 FF= 0.4000140E+01 F= 0.3243423E+00 Q= 0.4049586E+01

F
-0.3427129E 00 0.2398499E 00 0.7955879E 00 0.1886416E 00-0.9377459E 00
-0.3302187E 00 0.1621369E 00 0.7894699E 00-0.4096627E-03-0.2126188E 00
-0.3036758E 00 0.4692439E 00-0.1669025E-01-0.1821197E 00-0.1963136E 00
 0.6173351E 00-0.5551624E 00-0.4323723E 00 0.4281348E 00-0.1717609E 00

FM
  0.421   0.822   1.000   0.811   0.214   0.562   0.184   0.652   0.708   0.464
```

FP

-54.4	173.9	0.0	-121.0	70.5	-76.5	65.4	-91.8	18.5	138.7

TF

2.4116	0.0952	2.9077	-0.0472	2.6436	-0.3090	1.7975	-0.3829	0.9121	-0.0846
0.4927	0.3638	0.5556	0.5280	0.6876	0.2764	0.5985	-0.0896	0.4039	-0.2834
0.2961	-0.3522	0.2310	-0.4421	0.0865	-0.5259	-0.1097	-0.5108	-0.2429	-0.4192
-0.2599	-0.3459	-0.1844	-0.3355	-0.0742	-0.3590	-0.0032	-0.3730	-0.0323	-0.3757
-0.1496	-0.4062	-0.2566	-0.4862	-0.2469	-0.5705	-0.1000	-0.5771	0.1083	-0.4517
0.2513	-0.2044	0.2162	0.0656	0.0097	0.2043	-0.2235	0.1624	-0.3873	0.0718
-0.5540	0.0555	-0.7741	0.0575	-0.8721	-0.0190	-0.5656	-0.1191	0.2513	-0.1057
1.3812	0.0243								

TFM

2.4135	2.9081	2.6616	1.8379	0.9160	0.6125	0.7665	0.7411	0.6052	0.4934
0.4601	0.4988	0.5329	0.5225	0.4845	0.4326	0.3828	0.3666	0.3731	0.3771
0.4329	0.5497	0.6217	0.5857	0.4645	0.3240	0.2259	0.2046	0.2763	0.3939
0.5567	0.7[illegible]62	0.8723	0.5780	0.2727	1.3814				

VI. MAGNITUDE PATTERN SYNTHESIS WITH CONSTRAINED NORM

The program of this section implements the theory of sections IV and V, Report No. 2.

Punched card data is read in according to

```
      READ(1,30) N,M,NG,N9,BK,EPS
30    FORMAT(4I3, 2E14.7)
      READ(1,10)(X(I), I=1, N)
10    FORMAT(10F8.4)
      READ(1,10)(Y(I), I=1, N)
      READ(1,10)(PHI(I), I=1, M)
      READ(1,23)(AM(J), J=1, N)
23    FORMAT(5E14.7)
      NN = N*N
      READ(1,23)(P(J), J=1, NN)
      DO 22 JG=1, NG
      READ(1,23) CN
      READ(1,10)(G(I), I=1, M)
22    CONTINUE
```

The x and y coordinates of N point sources are read in as X and Y. The eigenvalues AM and eigenvectors P of the matrix $[\tilde{T}^*T]$ of (2-35) have been punched out by the program of section IV of this report. The starting value of $h_m e^{j\beta_m}$ in (2-21) is read in as G at M angles PHI in degrees in the plane of the point sources. The magnitude of G(m) is the magnitude of the specified pattern at the m^{th} angle. The phase portion $e^{j\beta_m}$ of G(m) implicitly gives the starting value for β_m. DO loop 22 makes it possible to deal successively with NG different specified patterns at the same propagation constant BK. The constraint C of (2-24) is read in as CN. The iterative process for finding the root of (2-45) is terminated as soon as $F(\alpha) \leq$ EPS*CN. Step 2 of the iteration procedure described in section IV, Report No. 2, is repeatedly executed until the pattern synthesis error ϵ of (2-19) ceases to decrease, but not more than a maximum of N9 times.

Minimum allocations are given by

```
COMPLEX P(N*N), T(N), TP(N*M), G(M), C(N),
    ALP(N), TF(M), F(N)
DIMENSION X(N), Y(N), PHI(M), AM(N),
    AM2(N), H(M), FF(N9+1), E(N9+1),
    Q(N9+1), CC(N), TFM(M), FM(N), FP(N)
```

DO loop 15 stores the K^{th} row of the matrix $[T]$ of (2-18) in T. DO loop 15 stores the K^{th} element of $[T]\vec{\phi}_J$ in TP((J-1)*M+K). DO loop 25 stores $\vec{h}$ of (2-19) in H, accumulates $||\vec{g}_o||^2$ of (2-14) in GG and replaces G by its complex conjugate.

Steps 1, 2, and 3 of the iteration procedure described in Section IV, Report No. 2, are executed for the $K9^{th}$ time inside DO loop 19. DO loop 18 stores C_J of (2-42) in C(J) and $|C_J|^2$ in CC(J). DO loop 31 stores F(0) of (2-45) in S6. If S6 < 0 DO loop 68 carries out Newton's method starting with α equal to the root of (2-45) for the previous value of K9. α is stored in both AL and AS(J9) while $F(\alpha)$ of (2-45) resides in FS(J9). Statement 60 calculates (1).

If $\alpha_{j+1} \leq 0$ then α_{j+1} is set equal to zero. The subscripted α's appearing in Equation (1) are not to be confused with those of (2-40). DO loop 91 puts α_i of (2-40) in ALP(I) and accumulates $||\vec{f}||^2$ in S5.

DO loop 92 stores the synthesized pattern

$$[T]\vec{f} = \sum_{i=1}^{N} \alpha_i [T]\vec{\phi}_i$$

in TF and its magnitude in TFM. DO loop 92 also accumulates $||[T]\vec{f}||^2$ and the pattern synthesis error ε of (2-23) in TFS and S1 respectively. Statement 56 changes the angle of G to that of $(T\vec{f})^*$. At the end of DO loop 19, the source norm squared

$$||\vec{f}||^2 = \sum_{i=1}^{N} |\alpha_i|^2 \ ,$$

the relative pattern synthesis error $\dfrac{\varepsilon}{||\vec{g}_o||^2}$, and Q defined by (2-15) as $M \dfrac{||\vec{f}||^2}{||[T]\vec{f}||^2}$ are stored in FF, E, and Q respectively.

DO loop 94 stores $\vec{f}$ in F and accumulates the source norm squared

$$||\vec{f}||^2 = \sum_{n=1}^{N} |f_n|^2$$

in S3. DO loop 94 disvovers that the J3th element of $\vec{f}$ is largest in magnitude. As expected, the second and fourth quantities S3 and S4 printed by statement 57 are very close to the final values of FF and Q. Statement 59 punches $\vec{f}$ on cards for possible input into the program of section IX of this report. DO loop 5ↄ normalizes the element of $\vec{f}$ largest in magnitude to 1 and then stores the magnitudes and phases in degrees of the elements of $\vec{f}$ in FM and FP respectively.

According to the printed outputs of the magnitude pattern synthesis with constrained source norm program of this section and the magnitude pattern synthesis program of section III of this report, it has been possible to reduce the source norm squared from 27.540 to about 8.0 with only a slight increase in the pattern synthesis error.

LISTING OF MAGNITUDE PATTERN SYNTHESIS WITH CONSTRAINED SOURCE NORM PROGRAM

```
//               (0034,FE,15S,1,4),'MAUTZ,JOE',REGION=200K
// EXEC WATFIV
//GO.FT02F001 DD SYSOUT=B,DCB=(RECFM=F,BLKSIZE=80)
//GO.SYSIN DD *
$JOB             MAUTZ,TIME=1,PAGES=40
      COMPLEX U,U1,CONJG,P(100),T(10),TP(360),G(36)
      COMPLEX C(10),ALP(10),TF(36),F(10)
      DIMENSION X(10),Y(10),PHI(36),AM(10),AM2(10),H(36),FF(61),E(61)
      DIMENSION Q(61),CC(10),AS(31),ES(30),TFM(36),FM(10),FP(10)
      PI=3.141593
      CP=PI/180.
      U=(0.,1.)
      READ(1,30) N,M,NG,N9,BK,EPS
   30 FORMAT(4I3,2E14.7)
      WRITE(3,32) N,M,NG,N9,BK,EPS
   32 FORMAT('0  N  M NG N9',6X,'BK',11X,'EPS'/1X,4I3,2E14.7)
      READ(1,10)(X(I),I=1,N)
   10 FORMAT(10F8.4)
      WRITE(3,13)(X(I),I=1,N)
   13 FORMAT('0X'/(1X,10F8.4))
      READ(1,10)(Y(I),I=1,N)
      WRITE(3,37)(Y(I),I=1,N)
   37 FORMAT('0Y'/(1X,10F8.4))
      READ(1,10)(PHI(I),I=1,M)
      WRITE(3,11)(PHI(I),I=1,M)
   11 FORMAT('0PHI'/(1X,10F8.4))
      READ(1,23)(AM(J),J=1,N)
   23 FORMAT(5E14.7)
      WRITE(3,24)(AM(J),J=1,N)
   24 FORMAT('0AM'/(1X,5E14.7))
      NN=N*N
      ZM=M
      JAM=(N+1)/2
      READ(1,23)(P(J),J=1,NN)
      WRITE(3,21)(P(J),J=1,NN)
   21 FORMAT('0P'/(1X,5E14.7))
      DO 29 J=1,N
      AM2(J)=AM(J)*AM(J)
   29 CONTINUE
      DO 14 K=1,M
      S1=PHI(K)*CP
      CS=COS(S1)
      SN=SIN(S1)
      DO 15 I=1,N
      S1=BK*(X(I)*CS+Y(I)*SN)
      T(I)=COS(S1)+U*SIN(S1)
   15 CONTINUE
      J1=0
      J2=K
      DO 16 J=1,N
      TP(J2)=0.
      DO 17 I=1,N
      J1=J1+1
      TP(J2)=TP(J2)+T(I)*P(J1)
   17 CONTINUE
      J2=J2+M
   16 CONTINUE
   14 CONTINUE
```

```
      DO 22 JG=1,NG
      READ(1,23) CN
      WRITE(3,28) CN
   28 FORMAT('0CN=',F14.7)
      READ(1,10)(G(I),I=1,M)
      WRITE(3,12)(G(I),I=1,M)
   12 FORMAT('0G'/(1X,10F8.4))
      EP=EPS*CN
      GG=0.
      DO 25 J=1,M
      U1=CONJG(G(J))
      S1=G(J)*U1
      GG=GG+S1
      H(J)=SQRT(S1)
      G(J)=U1
   25 CONTINUE
      FF(1)=0.
      F(1)=1.
      Q(1)=0.
      J9=1
      AL=AM(JAM)
      AS(1)=AL
  201 DO 19 K9=1,N9
      J1=0
      DO 18 J=1,N
      U1=0.
      DO 20 I=1,M
      J1=J1+1
      U1=U1+TP(J1)*G(I)
   20 CONTINUE
      C(J)=CONJG(U1)
      CC(J)=U1*C(J)
   18 CONTINUE
      S6=CN
      DO 31 K=1,N
      S6=S6-CC(K)/AM2(K)
   31 CONTINUE
      IF(S6) 66,65,65
   65 J9=1
      AL=0.
      AS(J9)=AL
      FS(J9)=S6
      GO TO 87
   66 AL=AS(J9)
      AS(1)=AL
      J9=1
      DO 68 J=1,30
      S2=CN
      S5=0.
      DO 67 K=1,N
      S3=AM(K)+AL
      S4=S3*S3
      S2=S2-CC(K)/S4
      S5=S5+CC(K)/(S4*S3)
   67 CONTINUE
      FS(J9)=S2
      IF(ABS(FS(J9)).LE.EP) GO TO 87
   60 AL=AS(J9)-.5*FS(J9)/S5
      J9=J9+1
      IF(AL.LE.0.) AL=0.
```

```
      AS(J9)=AL
   68 CONTINUE
      WRITE(3,219)
  219 FORMAT('ONEWTON' METHOD DID NOT CONVERGE')
   87 S5=0.
      DO 91 K=1,N
      ALP(K)=C(K)/(AM(K)+AL)
      S1=ALP(K)*CONJG(ALP(K))
      S5=S5+S1
   91 CONTINUE
      WRITE(3,90)(AS(K),K=1,J9)
   90 FORMAT(' CONVERGENCE OF LAGRANGE MULTIPLIER'/(1X,10E11.4))
      WRITE(3,100)(FS(K),K=1,J9)
  100 FORMAT(' FUNCTION'/(1X,10E11.4))
      S1=0.
      TFS=0.
      DO 92 J=1,M
      J2=J
      U1=0.
      DO 93 K=1,N
      U1=U1+TP(J2)*ALP(K)
      J2=J2+M
   93 CONTINUE
      TF(J)=U1
      U1=CONJG(U1)
      S2=TF(J)*U1
      TFS=TFS+S2
      TFM(J)=SQRT(S2)
      S2=TFM(J)-H(J)
      S1=S1+S2*S2
   56 G(J)=(H(J)/TFM(J))*U1
   92 CONTINUE
      J2=K9+1
      FF(J2)=S5
      F(J2)=S1/GG
      Q(J2)=ZM*S5/TFS
      IF(E(J2).GE.E(K9)) GO TO 44
   19 CONTINUE
   44 WRITE(3,51)(FF(I),I=1,J2)
   51 FORMAT('ONORM SQUARED OF F'/(1X,5E14.7))
      WRITE(3,52)(E(I),I=1,J2)
   52 FORMAT('ORELATIVE ERROR'/(1X,5E14.7))
      WRITE(3,53)(Q(I),I=1,J2)
   53 FORMAT('OQ'/(1X,5F14.7))
      S2=0.
      S3=0.
      DO 94 J=1,N
      F(J)=0.
      J1=J
      DO 95 I=1,N
      F(J)=F(J)+P(J1)*ALP(I)
      J1=J1+N
   95 CONTINUE
      S1=F(J)*CONJG(F(J))
      S3=S3+S1
      IF(S1.LT.S2) GO TO 94
      S2=S1
      J3=J
   94 CONTINUE
      U1=1./F(J3)
```

```
      S4=ZM*S3/TES
   57 WRITE(3,54) GG,S3,F(J2),S4
   54 FORMAT('0GG=',E14.7,' FF=',E14.7,' F=',E14.7,' Q=',E14.7)
      WRITE(3,48)(F(I),I=1,N)
   48 FORMAT('0F'/(1X,5E14.7))
   59 WRITE(2,23)(F(I),I=1,N)
      DO 55 J=1,N
      F(J)=U1*F(J)
      FM(J)=CABS(F(J))
      S1=REAL(F(J))
      S2=AIMAG(F(J))
      FP(J)=(ATAN2(S2,S1))/CP
   55 CONTINUE
      WRITE(3,42)(FM(J),J=1,N)
   42 FORMAT('0FM'/(1X,10F7.3))
      WRITE(3,43)(FP(J),J=1,N)
   43 FORMAT('0FP'/(1X,10F7.1))
      WRITE(3,49)(TE(K),K=1,M)
   49 FORMAT('0TE'/(1X,10F8.4))
      WRITE(3,50)(TEM(I),I=1,M)
   50 FORMAT('0TEM'/(1X,10F8.4))
   22 CONTINUE
      STOP
      END
$DATA
 10 36  1 40 0.2908882E+00 0.1000000E-02
  0.0000  5.3608 10.6048 15.5027 19.2648 19.2648 15.5027 10.6048  5.3608  0.0000
-10.0000 -9.6320 -8.4706 -6.2894 -2.5559  2.5559  6.2894  8.4706  9.6320 10.0000
  5.0000 15.0000 25.0000 35.0000 45.0000 55.0000 65.0000 75.0000 85.0000 95.0000
105.0000115.0000125.0000135.0000145.0000155.0000165.0000175.0000185.0000195.0000
205.0000215.0000225.0000235.0000245.0000255.0000265.0000275.0000285.0000295.0000
305.0000315.0000325.0000335.0000345.0000355.0000
 0.9467145E 02 0.8078329E 02 0.6195313E 02 0.5468497E 02 0.2875992E 02
 0.2551213E 02 0.9639283E 01 0.3275412E 01 0.5507557E 00 0.1628423E 00
 0.2903522E 00 0.2304621E-06 0.6844866E-01 0.4241972E-08-0.3647421E 00
-0.9729786E-07-0.4885511E 00-0.2973702E-07-0.1982093E 00 0.4377566E-06
 0.1982076E 00 0.2547125E-06 0.4885510E 00-0.3091387E-06 0.3647433E 00
-0.1850908E-06-0.6844884E-01 0.2713567E-06-0.2903540E 00 0.5429575E-06
-0.8914965E-01 0.2161319E-06 0.2720940E 00-0.1533020E-06 0.3308787E 00
-0.9951651E-07-0.1172659E 00 0.3858715E-06-0.5429362E 00 0.2947568E-06
-0.5429378E 00-0.5368720E-07-0.1172673E 00-0.5244318E-06 0.3308780E 00
 0.3087853E-06 0.2720941E 00 0.7309694E-06-0.8914953E-01 0.2100297E-06
 0.5298515E 00 0.4270248E-06 0.4308088E 00 0.1473907E-05 0.2920523E-01
 0.9519529E-06-0.7784194E-01-0.5524618E-06 0.1635162E 00-0.8845636E-06
 0.1635150E 00 0.1771575E-06-0.7784206E-01 0.4654154E-06 0.2921355E-01
-0.6627855E-06 0.4308203E 00-0.1429648E-05 0.5298546E 00-0.7521597E-06
-0.1047320E 00-0.1435871E-05-0.5457036E 00-0.1159389E-05-0.3948668E 00
 0.1412610E-06 0.8893049E-01 0.4475079E-06 0.1655635E 00-0.7249168E-06
-0.1655665E 00-0.8152254E-06-0.8892918E-01 0.5292864E-06 0.3948660E 00
 0.6803169E-06 0.5456957E 00-0.5820197E-06 0.1047221E 00-0.1504132E-05
-0.3773477E-01-0.2794079E-05 0.1875506E-01-0.4097609E-06-0.4167684E 00
 0.3626927E-05-0.5573131E 00 0.2356654E-05-0.1180260E 00-0.1477333E-05
-0.1180170E 00 0.3259473E-05-0.5573068E 00 0.5574124E-05-0.4167722E 00
 0.2242483E-05 0.1875898E-01 0.2413907E-06-0.3772328E-01 0.3268949E-05
-0.5359712E 00-0.3648856E-05-0.1195107E 00-0.8504847E-06 0.1060919E 00
-0.1918221E-05-0.2810081E 00-0.5109343E-05-0.3289738E 00-0.2711899E-05
 0.3289769E 00 0.1486194E-05 0.2810201E 00-0.1367136E-05-0.1060811E 00
-0.3287122E-05 0.1195102E 00 0.6517278E-06 0.5359692E 00 0.3198825E-05
-0.4139768E 00-0.4251949E-06 0.3121096E 00 0.3563546E-06 0.2024003E 00
 0.1425478E-06-0.1954105E 00-0.4762701E-06 0.3899477E 00 0.6987133E-06
```

```
 0.3899392E 00-0.3403692E-06-0.1954079E 00 0.5294441E-07 0.2024022E 00
 0.2056303E-06 0.3121062E 00 0.1403968E-06-0.4139754E 00-0.2686328E-06
 0.3210983E 00 0.4192697E-06-0.3446192E 00-0.4561953E-06 0.2366264E 00
 0.2257797E-06 0.1475003E 00-0.4949951E-06-0.4476349E 00-0.4168046E-07
 0.4476404E 00-0.4944204E-06-0.1475034E 00-0.3288654E-06-0.2366209E 00
 0.2878805E-06 0.3446209E 00-0.5696532E-06-0.3211029E 00 0.5917625E-06
 0.1962254E 00 0.1733023E-05-0.3776212E 00-0.3023024E-05 0.4183334E 00
 0.3710524E-05-0.3625054E 00-0.3121309E-05 0.1117757E 00 0.1425577E-05
 0.1117630E 00-0.4275416E-06-0.3624925E 00-0.8278869E-07 0.4183226E 00
 0.1338545E-06-0.3776149E 00-0.3584536E-06 0.1962231E 00 0.1241534E-07
-0.1198015E 00-0. 655849E-06 0.2539174E 00 0.4111487E-06-0.3791949E 00
-0.3516973E-06 0.3907360E 00-0.3590495E-06-0.3531159E 00 0.8053731E-06
 0.3531210E 00-0. 384469E-05-0.3907486E 00 0.2968422E-05 0.3792086E 00
-0.3526511E-05-0.2539279E 00 0.2444993E-05 0.1198068E 00-0.1146047E-05
 0.8000000E+01
  5.0191    0.0000   3.8637   0.0000   2.3662   0.0000   1.7434   0.0000   1.4142   0.0000
  1.2208    0.0000   1.1034   0.0000   1.0353   0.0000   1.0038   0.0000   0.0000   0.0000
  0.0000    0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000    0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000    0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000    0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000    0.0 00   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000    0.0000
$STOP
/*
//
```

PRINTED OUTPUT

```
  N  M NG N9       BK             EPS
 10 36  1 40 0.2908882E+00 0.1000000E-02

X
  0.0000   5.3608  10.6048  15.5027  19.2648  19.2648  15.5027  10.6048   5.3608   0.0000

Y
-10.0000  -9.6320  -8.4706  -6.2894  -2.5559   2.5559   6.2894   8.4706   9.6320  10.0000

PHI
  5.0000  15.0000  25.0000  35.0000  45.0000  55.0000  65.0000  75.0000  85.0000  95.0000
105.0000115.0000125.0000135.0000145.0000155.0000165.0000175.0000185.0000195.0000
205.0000215.0000225.0000235.0000245.0000255.0000265.0000275.0000285.0000295.0000
305.0000315.0000325.0000335.0000345.0000355.0000

AM
 0.9467145E+02 0.8078329E+02 0.6195313E+02 0.5468497E+02 0.2875992E+02
 0.2551213E+02 0.9639283E+01 0.3275412E+01 0.5507557E+00 0.1628423E+00

P
 0.2903522E+00 0.2304621E-06 0.6844866E-01 0.4241972E-08-0.3647421E+00
PLUS 39 MORE LINES OF PRINTED OUTPUT

CN= 0.8000000E+01

G
  5.0191    0.0000   3.8637   0.0000   2.3662   0.0000   1.7434   0.0000   1.4142   0.0000
  1.2208    0.0000   1.1034   0.0000   1.0353   0.0000   1.0038   0.0000   0.0000   0.0000
  0.0000    0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000    0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000    0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
```

```
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000
CONVERGENCE OF LAGRANGE MULTPLIER
 0.2876E+02 0.0000E+00 0.4080E-01 0.6232E-01 0.6618E-01
FUNCTION
 0.7731E+01-0.5365E+01-0.1474E+01-0.1977E+00-0.4759E-02
CONVERGENCE OF LAGRANGE MULTPLIER
 0.6618E-01 0.1016E+00 0.1116E+00 0.1121E+00
FUNCTION
-0.2261E+01-0.4185E+00-0.2132E-01-0.6294E-04
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1121E+00 0.1255E+00 0.1265E+00
FUNCTION
-0.5431E+00-0.3541E-01-0.1707E-03
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1265E+00 0.1239E+00
FUNCTION
 0.8753E-01-0.1105E-02
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1239E+00 0.1158E+00 0.1162E+00
FUNCTION
 0.2705E+00-0.1127E-01-0.1812E-04
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1162E+00 0.1273E+00 0.1280E+00
FUNCTION
-0.4284E+00-0.2316E-01-0.7629E-04
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1280E+00 0.1520E+00 0.1552E+00
FUNCTION
-0.9543E+00-0.1008E+00-0.1410E-02
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1552E+00 0.1672E+00 0.1678E+00
FUNCTION
-0.3975E+00-0.2006E-01-0.5817E-04
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1678E+00 0.1709E+00
FUNCTION
-0.9395E-01-0.1217E-02
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1709E+00 0.1713E+00
FUNCTION
-0.1031E-01-0.1907E-04
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1713E+00 0.1709E+00
FUNCTION
 0.1030E-01-0.1621E-04
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1709E+00 0.1705E+00
FUNCTION
 0.1159E-01-0.2003E-04
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1705E+00 0.1702E+00
FUNCTION
 0.8852E-02-0.1144E-04
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1702E+00
FUNCTION
 0.5974E-02
```

```
NORM SQUARED OF F
 0.0000000E+00 0.8004752E+01 0.8000054E+01 0.8000163E+01 0.8001095E+01
 0.8000013E+01 0.8000068E+01 0.8001402E+01 0.8000050E+01 0.8001208E+01
 0.8000008E+01 0.8000005E+01 0.8000011E+01 0.8000007E+01 0.7994021E+01

RELATIVE ERROR
 0.1000000E+01 0.2908099E+00 0.2666364E+00 0.2542084E+00 0.2456300E+00
 0.2330162E+00 0.2110200E+00 0.1959826E+00 0.1920178E+00 0.1912803E+00
 0.1911484E+00 0.1911200E+00 0.1911127E+00 0.1911109E+00 0.1911284E+00

0
 0.0000000E+00 0.7787708E+01 0.7495522E+01 0.7363955E+01 0.7257568E+01
 0.7140370E+01 0.6997487E+01 0.6878632E+01 0.6835937E+01 0.6829524E+01
 0.6826553E+01 0.6825109E+01 0.6823977E+01 0.6823167E+01 0.6817893E+01

GG= 0.5554509E+02 FF= 0.7993944E+01 F= 0.1911284E+00 0= 0.6817829E+01

F
-0.4177318E 00-0.4299659E-01 0.8461863E 00 0.4310746E 00-0.9412030E 00
-0.1100197E 01-0.9687752E-01 0.1416602E 01 0.5496796E 00-0.5293286E 00
-0.4408940E 00 0.4267602E 00 0.3108631E 00-0.4488897E 00-0.8891094E-01
 0.8172523E 00-0.3309910E 00-0.7269035E 00 0.4647018E 00 0.1289072E 00

FM
  0.290   0.65[illegible]   1.000   0.981   0.527   0.424   0.377   0.568   0.552   0.333

FP
  -43.6   157.[illegible]   -0.0 -135.5   86.6  -93.5   75.2 -133.2   16.1  146.1

TF
  2.8189   0.1368   3.2367  -0.3112   2.5793  -1.0900   0.9962  -1.4747  -0.6606  -0.904
 -1.3450   0.3585  -0.7852   1.3091   0.2037   1.2408   0.6494   0.5049   0.4381  -0.061
  0.1011  -0.1504  -0.0388  -0.0399  -0.0711   0.0096  -0.1257  -0.0149  -0.1940  -0.062
 -0.2238  -0.1152  -0.1922  -0.1438  -0.1138  -0.1183  -0.0407  -0.0568  -0.0265  -0.024
 -0.0681  -0.0637  -0.1186  -0.1425  -0.1557  -0.2002  -0.1729  -0.2330  -0.1268  -0.248
 -0.0183  -0.1366   0.0267  -0.0381  -0.0562   0.0483  -0.1277  -0.0039  -0.1062  -0.026
 -0.1623   0.1081  -0.4102   0.2456  -0.6066   0.2081  -0.3549   0.0903   0.4953   0.108
  1.7161   0.2472

TFM
  2.8257   3.2516   2.8002   1.7797   1.1202   1.[illegible]919   1.5265   1.2574   0.8225   0.442
  0.1812   0.0556   0.0718   0.1266   0.2040   0.2517   0.2401   0.1641   0.0699   0.035
  0.0932   0.1854   0.2537   0.2901   0.2791   0.1875   0.0466   0.0741   0.1278   0.10
  0.1950   0.4781   0.6413   0.3662   0.5071   1.7338
```

VII. PATTERN SYNTHESIS WITH CONSTRAINED QUALITY FACTOR

The program of this section is obtained by replacing the block of statements from (and including) statement 201 to statement 91 in the pattern synthesis with constrained source norm program of section V of this report by the block of statements listed in the present section. The punched card input data is the same as that of the pattern synthesis with constrained source norm program except that CN is interpreted not as the upper bound on the source norm squared but as the upper bound Q_o on the quality factor Q.

Summarizing the results of section VI, Report No. 2, the optimum source vector $\vec{f}$ is given by

$$\vec{f} = \left(1 + \beta \frac{Q_o}{M}\right) \sum_{i=1}^{N} \frac{C_i}{\lambda_i + \beta} \vec{\phi}_i \tag{2}$$

where Q_o is the upper bound on Q, C_i is given by (2-42) and λ_i and $\vec{\phi}_i$ satisfy (2-35). If

$$Q_o \geq M \frac{\sum_{i=1}^{N} \frac{|C_i|^2}{\lambda_i^2}}{\sum_{i=1}^{N} \frac{|C_i|^2}{\lambda_i}} = Q_3 \tag{3}$$

then $\beta = 0$. Otherwise, β is determined from

$$Q_o - M \frac{\sum_{i=1}^{N} \frac{|C_i|^2}{(\lambda_i+\beta)^2}}{\sum_{i=1}^{N} \frac{|C_i|^2 \lambda_i}{(\lambda_i+\beta)^2}} = F(\beta) = 0 \tag{4}$$

If

$$Q_o \le \frac{M}{\lambda_1} = Q_1 \tag{5}$$

where λ_1 is the largest eigenvalue of (2-35) then (4) has no solution because Q is always greater than Q_o. If

$$Q_1 \leq Q_o < M \frac{\sum_{i=1}^{N} |C_i|^2}{\sum_{i=1}^{N} |C_i|^2 \lambda_i} = Q_2 \tag{6}$$

then

$$-\infty < \beta \leq -\lambda_1 \tag{7}$$

If

$$Q_2 < Q_o < Q_3 \tag{8}$$

$$0 < \beta < \infty \tag{9}$$

The above scheme determines β uniquely because the left hand side of (4) is a monotone decreasing function of β in the intervals $(-\infty, -\lambda_1)$ and $(0\ \infty)$.

A verbal flow chart of the replacement block of statements is as follows. Just after DO loop 202, Q_2 and Q_3 of (6) and (3) are stored in Q2 and Q3 respectively. Starting with either $\beta = -\lambda_1 - \lambda_{(N+1)/2}$ or $\beta = \lambda_{(N+1)/2}$, DO loop 213 carries out Newton's method for finding the root of (4). According to Newton's method,

$$\beta_{i+1} = \beta_i - \frac{F(\beta_i)}{F'(\beta_i)} \tag{10}$$

Differentiating (4), we have

$$F'(\beta) = 2M \frac{\sum_{i=1}^{N} \frac{|C_i|^2}{(\lambda_i+\beta)^3} \sum_{i=1}^{N} \frac{|C_i|^2\lambda_i}{(\lambda_i+\beta)^2} - \sum_{i=1}^{N} \frac{|C_i|^2\lambda_i}{(\lambda_i+\beta)^3} \sum_{i=1}^{N} \frac{|C_i|^2}{(\lambda_i+\beta)^2}}{\left(\sum_{i=1}^{N} \frac{|C_i|^2\lambda_i}{(\lambda_i+\beta)^2}\right)^2} \tag{11}$$

An alternate form of expression (11) is

$$F'(\beta) = 2M \frac{\sum_{i=1}^{N} \frac{|c_i|^2}{(\lambda_i+\beta)^3} \sum_{i=1}^{N} \frac{|c_i|^2\lambda_i^2}{(\lambda_i+\beta)^3} - \left(\sum_{i=1}^{N} \frac{|c_i|^2\lambda_i}{(\lambda_i+\beta)^3}\right)^2}{\left(\sum_{i=1}^{N} \frac{|c_i|^2\lambda_i}{(\lambda_i+\beta)^2}\right)^2} \tag{12}$$

For large $|\beta|$, (11) is the difference of two terms each of which is proportional to $\frac{1}{\beta}$, but (12) is the difference of two terms each of which is proportional to $\frac{1}{\beta^2}$. Thus (12) is less susceptible to round off error when $|\beta|$ is large. However, both (11) and (12) are indeterminate when $\beta = -\lambda_1$. Equation (12) can be written as

$$F'(\beta) = \frac{2M}{\left(\sum_{i=1}^{N} \frac{|c_i|^2\lambda_i}{(\lambda_i+\beta)^2}\right)^2} \left(\left(\frac{|c_1|^2}{(\lambda_1+\beta)^3} + \sum_{i=2}^{N} \frac{|c_i|^2}{(\lambda_i+\beta)^3}\right) \right.$$

$$\left. \left(\frac{|c_1|^2\lambda_1^2}{(\lambda_1+\beta)^3} + \sum_{i=2}^{N} \frac{|c_i|^2\lambda_i^2}{(\lambda_i+\beta)^3}\right) - \left(\frac{|c_1|^2\lambda_1}{(\lambda_1+\beta)^3} + \sum_{i=2}^{N} \frac{|c_i|^2\lambda_i}{(\lambda_i+\beta)^3}\right)^2 \right) \tag{13}$$

After cancellation of the $\frac{1}{(\lambda_1+\beta)^6}$ terms, expression (13) becomes

$$F'(\beta) = \frac{2M}{\left(\sum_{i=1}^{N} \frac{|c_i|^2\lambda_i}{(\lambda_i+\beta)^2}\right)^2} \left(\sum_{i=2}^{N} \frac{|c_i|^2}{(\lambda_i+\beta)^3} \sum_{i=2}^{N} \frac{|c_i|^2\lambda_i^2}{(\lambda_i+\beta)^3} - \left(\sum_{i=2}^{N} \frac{|c_i|^2\lambda_i}{(\lambda_i+\beta)^3}\right)^2 \right.$$

$$\left. + \frac{|c_1|^2}{(\lambda_1+\beta)^3} \left(\sum_{i=2}^{N} \frac{|c_i|^2\lambda_i^2}{(\lambda_i+\beta)^3} + \lambda_1^2 \sum_{i=2}^{N} \frac{|c_i|^2}{(\lambda_i+\beta)^3} - 2\lambda_1 \sum_{i=2}^{N} \frac{|c_i|^2\lambda_i}{(\lambda_i+\beta)^3}\right) \right) \tag{14}$$

DO loop 214 accumulates in S1, S2, S3, S4, and S5 the five different sums appearing in (4) and (14). Next $F(\beta)$ of (4) is stored in FS and $F'(\beta)$ of (14) in DO. β of (10) is stored in both AL and AS. If the new β is outside the correct interval, the logic between statements 216 and 218 replaces β by the average of the previous β and the end point of the interval. DO loop 91 stores the coefficients of $\vec{\phi}_i$ appearing in (2) in ALP.

The sample punched card input data is numerically the same as that of the pattern synthesis with constrained source norm program except that CN = C.4000000 E+01 is replaced by CN = 0.4049586 E+01. The number 0.4049586 E+01 obtained from the printed output of the pattern synthesis with constrained source norm program is the Q of the source vector $\vec{f}$ which minimizes the pattern synthesis error subject to the constraint $||\vec{f}||^2 \leq 4$. Comparing printed outputs, we see that the pattern synthesis error of the pattern synthesis with constrained Q program is only slightly smaller than that of the pattern synthesis with constrained source norm program.

TO OBTAIN THE PATTERN SYNTHESIS WITH CONSTRAINED Q PROGRAM, REPLACE THE BLOCK OF STATEMENTS FROM STATEMENT 201 TO STATEMENT 91 IN THE PATTERN SYNTHESIS WITH CONSTRAINED SOURCE NORM PROGRAM BY THE FOLLOWING STATEMENTS.

```
      ZM2=2.*ZM
      AMM=-AM(1)
      Q1=ZM/AM(1)
      S1=0.
      S2=0.
      S3=0.
      S4=0.
      DO 202 K=1,N
      S1=S1+CC(K)/AM2(K)
      S2=S2+CC(K)/AM(K)
      S3=S3+CC(K)
      S4=S4+CC(K)*AM(K)
  202 CONTINUE
      Q2=ZM*S3/S4
      Q3=ZM*S1/S2
      WRITE(3,203) Q1,Q2,Q3
  203 FORMAT('0Q1=',E14.7,' Q2=',E14.7,' Q3=',E14.7)
      IF(CN.GE.Q1) GO TO 204
      WRITE(3,205)
  205 FORMAT('0SPECIFIED Q IS TOO SMALL')
      GO TO 22
  204 J9=1
      IF(CN.LE.Q3) GO TO 207
      AL=0.
      AS(J9)=AL
      FS(J9)=CN-Q3
      GO TO 208
  207 IF(CN.GT.Q2) GO TO 211
      KQ=1
      AL=-AM(1)-AM(JAM)
      GO TO 212
  211 KQ=2
      AL=AM(JAM)
  212 AS(J9)=AL
      DO 213 K=1,30
      S1=0.
      S2=0.
      S3=0.
      S4=0.
      S5=0.
      IF(N.EQ.1) GO TO 225
      DO 214 J=2,N
      S6=AM(J)+AL
      S7=S6*S6
      S8=CC(J)/S7
      S9=S8/S6
      S1=S1+S8
      S2=S2+S8*AM(J)
      S3=S3+S9
      S4=S4+S9*AM(J)
      S5=S5+S9*AM2(J)
  214 CONTINUE
  225 S6=AM(1)+AL
      S7=S6*S6
      S8=CC(1)/S7
```

```
      S1=S1+S8
      S2=S2+S8*AM(I)
      S9=S8/S6
      FS(J9)=CN-ZM*S1/S2
      IF(ABS(FS(J9)).LE.EP) GO TO 208
      DO=ZM2*(S3*S5-S4*S4+S9*(S5+AM2(I)*S3-2.*AM(I)*S4))/(S2*S2)
      AL=AL-FS(J9)/DO
      GO TO(216,217),KO
  216 IF(AL.LT.AMM) GO TO 218
      AL=.5*(ZMM+AS(J9))
      GO TO 218
  217 IF(AL.GT.0.) GO TO 218
      AL=.5*AS(J9)
  218 J9=J9+1
      AS(J9)=AL
  213 CONTINUE
      WRITE(3,219)
  219 FORMAT('ONEWTONS METHOD DID NOT CONVERGE')
  208 S3=1.+AL*CN/ZM
      DO 91 K=1,N
      ALP(K)= S3/(AM(K)+AL))*C(K)
   91 CONTINUE
```

PRINTED OUTPUT OF PATTERN SYNTHESIS WITH CONSTRAINED Q PROGRAM

```
   N   M NG        BK              EPS
  10  36  1 0.2908882E+00 0.1000000E-02

 X
   0.0000   5.3608 10.6048 15.5027 19.2648 19.2648 15.5027 10.6048  5.3608  0.000(

 Y
 -10.0000 -9.6320 -8.4706 -6.2894 -2.5559  2.5559  6.2894  8.4706  9.6320 10.000(

 PHI
   5.0000 15.0000 25.0000 35.0000 45.0000 55.0000 65.0000 75.0000 85.0000 95.000(
 105.0000115.0000125.0000135.0000145.0000155.0000165.0000175.0000185.0000195.000
 205.0000215.0000225.0000235.0000245.0000255.0000265.0000275.0000285.0000295.000
 305.0000315.0000325.0000335.0000345.0000355.0000

 AM
  0.9467145E+02 0.8078329E+02 0.6195313E+02 0.5468497E+02 0.2875992E+02
  0.2551213E+02 0.9639283E+01 0.3275412E+01 0.5507557E+00 0.1628423E+00

 P
  0.2903522E+00 0.2304621E-06 0.6844866E-01 0.4241972E-08-0.3647421E+00
 PLUS 39 MORE LINES OF PRINTED OUTPUT

 CN= 0.4049586E+01

 G
   5.0191   0.0000   3.8637   0.0000   2.3662   0.0000   1.7434   0.0000   1.4142   0.000
   1.2208   0.0000   1.1034   0.0000   1.0353   0.0000   1.0038   0.0000   0.0000   0.000
   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.000
   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.000
   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.000
   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.000
   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.000
   0.0000   0.0000
```

```
CC
 0.2552661E+03 0.4203154E+03 0.2834061E+02 0.2706128E+03 0.3613115E+03
 0.1311916E+03 0.1149906E+02 0.1260125E+02 0.2570739E+00 0.2755140E+00

Q1= 0.3802624E+00 Q2= 0.6063326E+00 Q3= 0.1259200E+02
CONVERGENCE OF LAGRANGE MULTIPLIER
 0.2876E+02 0.1438E+02 0.7190E+01 0.3595E+01 0.1797E+01 0.8987E+00 0.4494E+00
 0.1079E+00 0.1878E+00 0.2355E+00 0.2460E+00
FUNCTION
 0.3182E+01 0.3068E+01 0.2913E+01 0.2686E+01 0.2364E+01 0.1895E+01 0.1116E+01
-0.2251E+01-0.6632E+00-0.1034E+00-0.3542E-02

GG= 0.5554509E+02 FF= 0.4228370E+01 F= 0.3238363E+00 Q= 0.4052990E+01

F
-0.3523602E 00 0.2463782E 00 0.8179110E 00 0.1941064E 00-0.9640898E 00
-0.3397281E 00 0.1669634E 00 0.8117120E 00-0.6355643E-03-0.2188625E 00
-0.3119421E 00 0.4825922E 00-0.1727355E-01-0.1875600E 00-0.2016132E 00
 0.6348063E 00-0.5706120E 00-0.4445961E 00 0.4400374E 00-0.1763967E 00

FM
  0.421   0.822   1.000   0.811   0.214   0.562   0.184   0.652   0.708   0.464

FP
  -54.4  173.9     0.0 -121.0    70.4   -76.5    65.3   -91.8    18.5   138.7

TF
  2.4785  0.0978  2.9884 -0.0485  2.7169 -0.3176  1.8472 -0.3935  0.9372 -0.0868
  0.5064  0.3740  0.5711  0.5426  0.7067  0.2839  0.6150 -0.0921  0.4150 -0.2912
  0.3043 -0.3618  0.2374 -0.4543  0.0888 -0.5405 -0.1129 -0.5250 -0.2497 -0.4307
 -0.2671 -0.3555 -0.1894 -0.3448 -0.0762 -0.3690 -0.0033 -0.3834 -0.0332 -0.3860
 -0.1538 -0.4174 -0.2637 -0.4997 -0.2536 -0.5864 -0.1027 -0.5931  0.1114 -0.4642
  0.2583 -0.2101  0.2222  0.0673  0.0100  0.2100 -0.2297  0.1669 -0.3980  0.0737
 -0.5693  0.0569 -0.7955  0.0591 -0.8963 -0.0196 -0.5813 -0.1223  0.2582 -0.1086
  1.4194  0.0250

TFM
  2.4804  2.9888  2.7354  1.8887  0.9412  0.6295  0.7878  0.7616  0.6218  0.5069
  0.4728  0.5126  0.5477  0.5370  0.4979  0.4446  0.3935  0.3768  0.3834  0.3874
  0.4448  0.5650  0.6389  0.6019  0.4774  0.3330  0.2322  0.2102  0.2839  0.4048
  0.5721  0.7977  0.8965  0.5941  0.2801  1.4196
```

VIII. MAGNITUDE PATTERN SYNTHESIS WITH CONSTRAINED QUALITY FACTOR

The program of this section is obtained by replacing the block of statements from (and including) statement 201 to statement 91 in the magnitude pattern synthesis with constrained source norm program of section VI of this report by the block of statemetns listed in the present section. The punched card input data is the same as that of the magnitude pattern synthesis with constrained source norm program except that CN is interpreted not as the upper bound on the source norm squared but as the upper bound Q_o on the quality factor Q.

A verbal flow chart of the replacement block of statements is as follows. Steps 1, 2, and 3 of the iteration procedure described in section IV, Report No. 2 are executed for the $K9^{th}$ time inside DO loop 19. DO loop 18 stores C_J of (2-42) in C(J) and $||C_J||^2$ in CC(J). Just after DO loop 202, Q_2 and Q_3 of (6) and (7) are stored in Q2 and Q3 respectively. The logic between statements 203 and 212 adjusts the starting value of β appearing in (4) if it is not in the correct interval. β is stored in both AL and AS. For K9=1, the unadjusted starting value of β is $\lambda_{(N+1)/2}$. For K9 > 1, the unadjusted starting value of β is the value of β calculated from the scheme (3)-(9) for the previous value of K9. DO loop 213 carries out Newton's method for finding the root of (4). DO loop 214 accumulates in S1, S2, S3, S4, and S5 the five different sums appearing in (4) and (14). Next, $F(\beta)$ of (4) is stored in FS and $F'(\beta)$ of (14) in DQ. If the new β calculated from (10) is outside the correct interval, the logic between statements 216 and 218 replaces β by the average of the previous β and the end point of the interval. DO loop 91 stores the coefficients of $\dot{\phi}_i$ appearing in (2) in ALP.

The sample punched card input data is numerically the same as that of the magnitude pattern synthesis with constrained source norm program except that CN = 0.8000000 E+01 is replaced by CN = 0.6817829 E+01. The number 0.6817829 E+01 obtained from the printed output of the magnitude pattern synthesis with constrained source norm program is the Q of the source vector $\dot{f}$ which minimizes the pattern synthesis error subject to the constraint $||\vec{f}||^2 \leq 8$. Comparing printed outputs, the pattern synthesis error of the magnitude pattern synthesis with constrained Q program is only slightly smaller than that of the magnitude pattern synthesis with constrained source norm program.

TO OBTAIN THE MAGNITUDE PATTERN SYNTHESIS WITH CONSTRAINED Q PROGRAM, REPLACE THE BLOCK OF STATEMENTS FROM STATEMENT 201 TO STATEMENT 91 IN THE MAGNITUED PATTERN SYNTHESIS WITH CONSTRAINED SOURCE NORM PROGRAM BY THE FOLLOWING STATEMENTS.

```
      AMM=-AM(1)
      ZM2=2.*ZM
      Q1=ZM/AM(1)
      IF(CN.GE.Q1) GO TO 204
      WRITE(3,205)
  205 FORMAT('OSPECIFIED Q IS TOO SMALL')
      GO TO 22
  204 DO 19 K9=1,N9
      J1=0
      DO 18 J=1,N
      U1=0.
      DO 20 I=1,M
      J1=J1+1
      U1=U1+TP(J1)*G(I)
   20 CONTINUE
      C(J)=CONJG(U1)
      CC(J)=U1*C(J)
   18 CONTINUE
      S1=0.
      S2=0.
      S3=0.
      S4=0.
      DO 202 K=1,N
      S1=S1+CC(K)/AM2(K)
      S2=S2+CC(K)/AM(K)
      S3=S3+CC(K)
      S4=S4+CC(K)*AM(K)
  202 CONTINUE
      Q2=ZM*S3/S4
      Q3=ZM*S1/S2
      WRITE(3,203) Q1,Q2,Q3
  203 FORMAT('0Q1=',E14.7,' Q2=',E14.7,' Q3=',E14.7)
      J9=1
      IF(CN.LE.Q3) GO TO 207
      AL=0.
      AS(J9)=AL
      FS(J9)=CN-Q3
      GO TO 208
  207 IF(CN.GT.Q2) GO TO 211
      KO=1
      IF(AL.GT.AMM) AL=-AM(1)-AM(JAM)
      GO TO 212
  211 KO=2
      IF(AL.LT.0.) AL=AM(JAM)
  212 AS(J9)=AL
      DO 213 K=1,30
      S1=0.
      S2=0.
      S3=0.
      S4=0.
      S5=0.
      IF(N.EQ.1) GO TO 225
      DO 214 J=2,N
      S6=AM(J)+AL
      S7=S6*S6
```

```
      S8=CC(J)/S7
      S9=S8/S6
      S1=S1+S8
      S2=S2+S8*AM(J)
      S3=S3+S9
      S4=S4+S9*AM(J)
      S5=S5+S9*AM2(J)
  214 CONTINUE
  225 S6=AM(1)+AL
      S7=S6*S6
      S8=CC(1)/S7
      S1=S1+S8
      S2=S2+S8*AM(1)
      S9=S8/S6
      FS(J9)=CN-ZM*S1/S2
      IF(ABS(FS(J9)).LE.EP) GO TO 208
      DQ=ZM2*(S3*S5-S4*S4+S9*(S5+AM2(1)*S3-2.*AM(1)*S4))/(S2*S2)
      AL=AL-FS(J9)/DQ
      GO TO (216,217),KQ
  216 IF(AL.LT.AMM) GO TO 218
      AL=.5*(AMM+AS(J9))
      GO TO 218
  217 IF(AL.GT.0.) GO TO 218
      AL=.5*AS(J9)
  218 J9=J9+1
      AS(J9)=AL
  213 CONTINUE
      WRITE(3,219)
  219 FORMAT('ONEWTONS METHOD DID NOT CONVERGE')
  208 S3=1.+AL*CN/ZM
      S5=0.
      DO 91 K=1,N
      ALP(K)=(S3/(AM(K)+AL))*C(K)
      S4=ALP(K)*CONJG(ALP(K))
      S5=S5+S4
   91 CONTINUE
```

PRINTED OUTPUT OF MAGNITUDE PATTERN SYNTHESIS WITH CONSTRAINED Q PROGRAM

```
  N  M NG N9       BK              EPS
 10 36  1 40 0.2908882E+00 0.1000000E-02

X
  0.0000  5.3608 10.6048 15.5027 19.2648 19.2648 15.5027 10.6048  5.3608  0.0000

Y
-10.0000 -9.6320 -8.4706 -6.2894 -2.5559  2.5559  6.2894  8.4706  9.6320 10.0000

PHI
  5.0000 15.0000 25.0000 35.0000 45.0000 55.0000 65.0000 75.0000 85.0000 95.0000
105.0000115.0000125.0000135.0000145.0000155.0000165.0000175.0000185.0000195.0000
205.0000215.0000225.0000235.0000245.0000255.0000265.0000275.0000285.0000295.0000
305.0000315.0000325.0000335.0000345.0000355.0000

AM
 0.9467145E+02 0.8078329E+02 0.6195313E+02 0.5468497E+02 0.2875992E+02
 0.2551213E+02 0.9639283E+01 0.3275412E+01 0.5507557E+00 0.1628423E+00

P
 0.2903522E+00 0.2304621E-06 0.6844866E-01 0.4241972E-08-0.3647421E+00
```

```
PLUS 39 MORE LINES OF PRINTED OUTPUT

CN= 0.6817829E+01

G
  5.0191  0.0000  3.8637  0.0000  2.3662  0.0000  1.7434  0.0000  1.4142  0.0000
  1.2208  0.0000  1.1034  0.0000  1.0353  0.0000  1.0038  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000

Q1= 0.3802624E+00 Q2= 0.6063330E+00 Q3= 0.1259209E+02
CONVERGENCE OF LAGRANGE MULTPLIER
 0.2876E+02 0.1438E+02 0.7190E+01 0.3595E+01 0.1797E+01 0.8987E+00 0.4494E+00
 0.2247E+00 0.1123E+00 0.8865E-01 0.9111E-01
FUNCTION
 0.5951E+01 0.5836E+01 0.5681E+01 0.5455E+01 0.5133E+01 0.4663E+01 0.3885E+01
 0.2554E+01 0.6388E+00-0.8500E-01-0.1143E-02

Q1= 0.3802624E+00 Q2= 0.6264157E+00 Q3= 0.1512583E+02
CONVERGENCE OF LAGRANGE MULTPLIER
 0.9111E-01 0.1246E+00 0.1319E+00 0.1322E+00
FUNCTION
-0.1360E+01-0.2119E+00-0.7212E-02-0.5722E-05

Q1= 0.3802624E+00 Q2= 0.6252590E+00 Q3= 0.1593605E+02
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1322E+00 0.1435E+00 0.1441E+00
FUNCTION
-0.3209E+00-0.1624E-01-0.4292E-04

Q1= 0.3802624E+00 Q2= 0.6142960E+00 Q3= 0.1556036E+02
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1441E+00 0.1380E+00
FUNCTION
 0.1508E+00-0.4280E-02

Q1= 0.3802624E+00 Q2= 0.5984214E+00 Q3= 0.1473095E+02
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1380E+00 0.1263E+00 0.1269E+00
FUNCTION
 0.2864E+00-0.1629E-01-0.4482E-04

Q1= 0.3802624E+00 Q2= 0.5790257E+00 Q3= 0.1525220E+02
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1269E+00 0.1350E+00 0.1353E+00
FUNCTION
-0.2290E+00-0.8540E-02-0.1335E-04

Q1= 0.3802624E+00 Q2= 0.5629661E+00 Q3= 0.1689378E+02
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1353E+00 0.1555E+00 0.1576E+00
FUNCTION
-0.5969E+00-0.5125E-01-0.4587E-03

Q1= 0.3802624E+00 Q2= 0.5541689E+00 Q3= 0.17744440E+02
CONVERGENCE OF LAGRANGE MULTPLIER
```

```
 0.1576E+00 0.1678E+00 0.1683E+00
FUNCTION
-0.2614E+00-0.1090E-01-0.2193E-04

Q1= 0.3802624E+00 Q2= 0.5500408E+00 Q3= 0.1801064E+02
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1683E+00 0.1711E+00
FUNCTION
-0.6826E-01-0.8011E-03

Q1= 0.3802624E+00 Q2= 0.5481082E+00 Q3= 0.1807091E+02
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1711E+00 0.1714E+00
FUNCTION
-0.7845E-02-0.1717E-04

Q1= 0.3802624E+00 Q2= 0.5471681E+00 Q3= 0.1807094E+02
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1714E+00 0.1711E+00
FUNCTION
 0.8230E-02-0.1621E-04

Q1= 0.3802624E+00 Q2=0.5466917E+00 Q3= 0.1805829E+02
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1711E+00 0.1707E+00
FUNCTION
 0.9362E-02-0.1621E-04


Q1= 0.3802624E+00 Q2= 0.5464411E+00 Q3= 0.1804625E+02
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1707E+00 0.1704E+00
FUNCTION
 0.7169E-02-0.7629E-05

Q1= 0.3802624E+00 Q2= 0.5463050E+00 Q3= 0.1803748E+02
CONVERGENCE OF LAGRANGE MULTPLIER
 0.1704E+00
FUNCTION
 0.4828E-02

NORM SQUARED OF F
 0.0000000E+00 0.7195872E+01 0.7600381E+01 0.7780138E+01 0.7892490E+01
 0.7994216E+01 0.8195385E+01 0.8413612E+01 0.8496351E+01 0.8514475E+01
 0.8516630E+01 0.8517226E+01 0.8517363E+01 0.8517389E+01 0.8511551E+01

RELATIVE ERROR
 0.1000000E+01 0.2924485E+00 0.2678614E+00 0.2548482E+00 0.2456651E+00
 0.2321668E+00 0.2096755E+00 0.1949906E+00 0.1912015E+00 0.1905022E+00
 0.1903772E+00 0.1903507E+00 0.1903440E+00 0.1903419E+00 0.1903595E+00

Q
 0.0000000E+00 0.6818702E+01 0.6817563E+01 0.6817606E+01 0.6821848E+01
 0.6817612E+01 0.6817593E+01 0.6818030E+01 0.6817611E+01 0.6818388E+01
 0.6817616E+01 0.6817608E+01 0.6817613E+01 0.6817599E+01 0.6812769E+01

GG= 0.5554509E+02 FF= 0.8511467E+01 E= 0.1903595E+00 Q= 0.6812698E+01

F
-0.4309598E+00-0.4540056E-01 0.8721079E+00 0.4469206E+00-0.9683221E+00
```

```
-0.1137715E+01-0.1040341E+00 0.1461415E+01 0.5688261E+00-0.5443448E+00
-0.4561355E+00 0.4389312E+00 0.3219826E+00-0.4621358E+00-0.9385562E-01
 0.8430707E+00-0.3397757E+00-0.7510453E+00 0.4793990E+00 0.1342400E+00

FM
  0.290   0.656   1.000   0.981   0.527   0.424   0.377   0.568   0.552   0.333

FP
  -43.6   157.5     0.0 -135.5    86.7   -93.5    75.3 -133.2    16.1   146.0

TF
  2.9092  0.2108  3.3419 -0.3124  2.6655 -1.1181  1.0326 -1.5195 -0.6792 -0.9359
 -1.3893  0.3660 -0.8144  1.3489  0.2065  1.2814  0.6688  0.5232  0.4525 -0.0625
  0.1049 -0.1550 -0.0400 -0.0413 -0.0735  0.0098 -0.1298 -0.0157 -0.2001 -0.0654
 -0.2308 -0.1195 -0.1982 -0.1489 -0.1172 -0.1224 -0.0419 -0.0587 -0.0272 -0.0250
 -0.0701 -0.0659 -0.1221 -0.1474 -0.1604 -0.2071 -0.1780 -0.2410 -0.1303 -0.2570
 -0.0185 -0.1927  0.0276 -0.0393 -0.0583  0.0496 -0.1318 -0.0044 -0.1096 -0.0273
 -0.1678  0.1114 -0.4242  0.2525 -0.6267  0.2133 -0.3665  0.0923  0.5111  0.1132
  1.7708  0.2597

TFM
  2.9168  3.3564  2.8905  1.8371  1.1564  1.4367  1.5757  1.2980  0.8491  0.4567
  0.1872  0.0575  0.0742  0.1307  0.2105  0.2599  0.2479  0.1695  0.0721  0.0370
  0.0962  0.1914  0.2619  0.2996  0.2882  0.1936  0.0480  0.0765  0.1319  0.1129
  0.2014  0.4937  0.6620  0.3780  0.5235  1.7897
```

IX. COMPUTATION OF SYNTHESIZED PATTERNS FROM THE SOURCE VECTOR

The program of this section stores the synthesized patterns on a direct access data set so that they may be plotted by the program of section X of this report.

The activity on data sets 1 (punched card input) and 6 (direct access input and output) is as follows.

```
      READ(1,14) N6, N, NK, NP
14    FORMAT(20I3)
      READ(1,16)(BK(I), I=1, NK)
16    FORMAT(5E14.7)
      READ(1,14)(NG(I), I=1, NK)
      READ(1,18)(X(I), I=1, N)
18    FORMAT(10F8.4)
      READ(1,18)(Y(I), I=1, N)
      REWIND 6
      IF(N6) 11, 11, 12
12    DO 13 J=1, N6
      READ (6)
13    CONTINUE
11    FN = NP-1
      DO 25 JK=1, NK
      KG = NG(JK)
      DO 29 JG = 1, KG
      READ(1,16)(F(I), I=1, N)
29    CONTINUE
25    CONTINUE
      WRITE(6)(TFM(I), I=1, J9)
```

The x and y coordinates of N point sources are read in as X and Y. A source vector $\vec{f}$ at propagation constant BK(JK) is read in through F. Inside DO loop 29 the synthesized pattern corresponding to the source vector F is calculated as the magnitude of expression (2-17) evaluated at $\phi = 0, \Delta\phi, 2\Delta\phi\ldots$ in radians where $\Delta\phi = \frac{2\pi}{NP-1}$. The synthesized patterns are stored one after the other in TFM.

Minimum allocations are given by

```
COMPLEX T(N*NP), F(N)
DIMENSION BK(NK), NG(NK), X(N), Y(N)
    XCS(N*NP), TFM(J9)
```

where

$$J9 = NP* \sum_{I=1}^{NK} NG(I)$$

Nested DO loops 23 and 24 put $x_I \cos\phi + y_I \sin\phi$ of (2-17) in XCS. The index JK of DO loop 25 denotes propagation constant BK(JK). Nested DO loops 26 and 27 store the matrix [T] of (2-18) by rows in T . The index JG of DO loop 29 denotes the JGth synthesized pattern to be computed at propagation constant BK(JK). Nested DO loops 31 and 32 store the synthesized pattern $\|[T]\acute{f}\|$ of (2-17) in TFM.

The sample punched card input data is such that the patterns synthesized by the programs of sections II, III, V, VI, VII, and VIII, of this report are evaluated at every 2-1/2 degrees and stored on the first record of data set 6.

LISTING OF PROGRAM TO COMPUTE SYNTHESIZED PATTERNS FROM SOURCE VECTOR F

```
//               (0034,EE,15S,1),'MAUTZ,JOE',REGION=200K
// EXEC WATFIV
//GO.FT06F001 DD DSNAME=EE0034.REV1,DISP=OLD,UNIT=3330,                X
//               VOLUME=SER=SU0009,DCB=(RECFM=VS,BLKSIZE=2596,LRECL=2592,X
//               BUFNO=1)
//GO.SYSIN DD *
$JOB             MAUTZ,TIME=1,PAGES=40
      COMPLEX U,U1,T(1450),F(10)
      DIMENSION BK(10),NG(10),X(10),Y(10),XCS(1450),TEM(2900)
      U=(0.,1.)
      PI=3.141593
      READ(1,14) N6,N,NK,NP
   14 FORMAT(20I3)
      WRITE(3,15) N6,N,NK,NP
   15 FORMAT('0 N6   N NK NP'/1X,4I3)
      READ(1,16)(BK(I),I=1,NK)
   16 FORMAT(5E14.7)
      WRITE(3,17)(BK(I),I=1,NK)
   17 FORMAT('0BK'/(1X,5E14.7))
      READ(1,14)(NG(I),I=1,NK)
      WRITE(3,28)(NG(I),I=1,NK)
   28 FORMAT('0NG'/(1X,20I3))
      READ(1,18)(X(I),I=1,N)
   18 FORMAT(10F8.4)
      WRITE(3,19)(X(I),I=1,N)
   19 FORMAT('0X'/(1X,10F8.4))
      READ(1,18)(Y(I),I=1,N)
      WRITE(3,20)(Y(I),I=1,N)
   20 FORMAT('0Y'/(1X,10F8.4))
      REWIND 6
      IF(N6) 11,11,12
   12 DO 13 J=1,N6
      READ(6)
   13 CONTINUE
   11 FN=NP-1
      DEL=2.*PI/FN
      J1=0
      DO 23 J=1,NP
      S1=(J-1)*DEL
      CS=COS(S1)
      SN=SIN(S1)
      DO 24 I=1,N
      J1=J1+1
      XCS(J1)=X(I)*CS+Y(I)*SN
   24 CONTINUE
   23 CONTINUE
      J9=0
      DO 25 JK=1,NK
      KG=NG(JK)
      BB=BK(JK)
      J1=0
      DO 26 J=1,NP
      DO 27 I=1,N
      J1=J1+1
      S1=BB*XCS(J1)
      T(J1)=COS(S1)+U*SIN(S1)
   27 CONTINUE
   26 CONTINUE
```

```
      DO 29 JG=1,KG
      READ(1,16)(F(I),I=1,N)
      WRITE(3,30)(F(I),I=1,N)
   30 FORMAT('OF'/(1X,5F14.7))
      J1=0
      DO 31 J=1,NP
      U1=0.
      DO 32 I=1,N
      J1=J1+1
      U1=U1+T(J1)*F(I)
   32 CONTINUE
      J9=J9+1
      TFM(J9)=CABS(U1)
   31 CONTINUE
   29 CONTINUE
   25 CONTINUE
      WRITE(6)(TFM(I),I=1,J9)
      N9=J9/NP
      J1=1
      DO 33 J=1,N9
      J2=J1+9
      WRITE(3,35) J
   35 FORMAT('OFIRST 10 POINTS ON THE',I3,' TH PATTERN')
      WRITE(3,34)(TFM(I),I=J1,J2)
   34 FORMAT(1X,10F8.4)
      J1=J1+NP
   33 CONTINUE
      STOP
      END
$DATA
  0 10  1145
 0.2908882E+00
  6
  0.0000  5.3608 10.6048 15.5027 19.2648 19.2648 15.5027 10.6048  5.3608  0.0000
-10.0000 -9.6320 -8.4706 -6.2894 -2.5559  2.5559  6.2894  8.4706  9.6320 10.0000
-0.5421209E+00 0.5224890E-01 0.1187810E+01 0.5969831E+00-0.1467048E+01
-0.9448900E+00 0.6552452E+00 0.1432544E+01-0.3653175E+00-0.7956181E+00
-0.9229600E-02 0.1055890E+01-0.3044195E+00-0.8306894E+00 0.8837318E-01
 0.1247218E+01-0.7381020E+00-0.8539429E+00 0.5231936E+00 0.2333546E-01
-0.6131346E+00-0.2752376E+00 0.1244744E+01 0.9962800E+00-0.1532891E+01
-0.1988470E+01 0.4787318E+00 0.2502643E+01 0.2811250E+00-0.1595194E+01
-0.3373244E+00 0.1369034E+01 0.2236381E+00-0.1435582E+01 0.1607297E-01
 0.1691813E+01-0.4139780E+00-0.1279405E+01 0.4812400E+00 0.3546119E+00
-0.3427129E+00 0.2398499E+00 0.7955879E+00 0.1886416E+00-0.9377459E+00
-0.3302187E+00 0.1621369E+00 0.7894699E+00-0.4096627E-03-0.2126188E+00
-0.3036758E+00 0.4692439E+00-0.1669025E-01-0.1821197E+00-0.1963136E+00
 0.6173351E+00-0.5551624E+00-0.4323723E+00 0.4281348E+00-0.1717609E+00
-0.4177318E+00-0.4299659E-01 0.8461863E+00 0.4310746E+00-0.9412030E+00
-0.1100197E+01-0.9687752E-01 0.1416602E+01 0.5496796E+00-0.5293286E+00
-0.4408940E+00 0.4267602E+00 0.3108631E+00-0.4488897E+00-0.8891094E-01
 0.8172523E+00-0.3309910E+00-0.7269035E+00 0.4647018E+00 0.1289072E+00
-0.3523602E+00 0.2463782E+00 0.8179110E+00 0.1941064E+00-0.9640898E+00
-0.3397281E+00 0.1669634E+00 0.8117120E+00-0.6355643E-03-0.2188625E+00
-0.3119421E+00 0.4825922E+00-0.1727355E-01-0.1875600E+00-0.2016132E+00
 0.6348063E+00-0.5706120E+00-0.4445961E+00 0.4400374E+00-0.1763967E+00
-0.4309598E 00-0.4540056E-01 0.8721079E 00 0.4469206E 00-0.9683221E 00
-0.1137715E 01-0.1040341E 00 0.1461415E 01 0.5688261E 00-0.5443448E 00
-0.4561355E 00 0.4389312E 00 0.3219826E 00-0.4621358E 00-0.9385562E-01
 0.8430707E 00-0.3397757E 00-0.7510453E 00 0.4793990E 00 0.1342400E 00
$STOP
```

```
/*
//

PRINTED OUTPUT

 N6   N NK NP
  0 10  1145

BK
 0.2908882E+00

NG
   6

X
  0.0000  5.3608 10.6048 15.5027 19.2648 19.2648 15.5027 10.6048  5.3608  0.0000

Y
-10.0000 -9.6320 -8.4706 -6.2894 -2.5559  2.5559  6.2894  8.4706  9.6320 10.0000

F
-0.5421209E+00 0.5224890E-01 0.1187810E+01 0.5969831E+00-0.1467048E+01
-0.9448900E+00 0.6552452E+00 0.1432544E+01-0.3653175E+00-0.7956181E+00
-0.9229600E-02 0.1055890E+01-0.3044195E+00-0.8306894E+00 0.8837318E-01
 0.1247218E+0 -0.7381020E+00-0.8539429E+00 0.5231936E+00 0.2333546E-01

F
-0.6131346E+00-0.2752376E+00 0.1244744E+01 0.9962800E+00-0.1532891E+01
-0.1988470E+0  0.4787318E+00 0.2502643E+01 0.2811250E+00-0.1595194E+01
-0.3373244E+00 0.1369034E+01 0.2236381E+00-0.1435582E+01 0.1607297E-01
 0.1691813E+0 -0.4139780E+00-0.1279405E+01 0.4812400E+00 0.3546119E+00

F
-0.3427129E+00 0.2398499E+00 0.7955879E+00 0.1886416E+00-0.9377459E+00
-0.3302187E+00 0.1621369E+00 0.7894699E+00-0.4096627E-03-0.2126188E+00
-0.3036758E+00 0.4692439E+00-0.1669025E-01-0.1821197E+00-0.1963136E+00
 0.6173351E+00-0.5551624E+00-0.4323723E+00 0.4281348E+00-0.1717609E+00

F
-0.4177318E+00-0.4299659E-01 0.8461863E+00 0.4310746E+00-0.9412030E+00
-0.1100197E+0 -0.9687752E-01 0.1416602E+01 0.5496796E+00-0.5293286E+00
-0.4408940E+00 0.4267602E+00 0.3108631E+00-0.4488897E+00-0.8891094E-01
 0.8172523E+0)-0.3309910E+00-0.7269035E+00 0.4647018E+00 0.1289072E+00

F
-0.3523602E+00 0.2463782E+00 0.8179110E+00 0.1941064E+00-0.9640898E+00
-0.3397281E+0) 0.1669634E+00 0.8117120E+00-0.6355643E-03-0.2188625E+00
-0.3119421E+00 0.4825922E+00-0.1727355E-01-0.1875600E+00-0.2016132E+00
 0.6348063E+0)-0.5706120E+00-0.4445961E+00 0.4400374E+00-0.1763967E+00

F
-0.4309598E 00-0.4540056E-01 0.8721079E 00 0.4469206E 00-0.9683221E 00
-0.1137715E 01-0.1040341E 00 0.1461415E 01 0.5688261E 00-0.5443448E 00
-0.4561355E 0) 0.4389312E 00 0.3219826E 00-0.4621358E 00-0.9385562E-01
 0.8430707E 0)-0.3397757E 00-0.7510453E 00 0.4793990E 00 0.1342400E 00

FIRST 10 POINTS ON THE  1 TH PATTERN
  1.9967  2.2396  2.5691  2.7961  2.9730  3.0940  3.1553  3.1555  3.0953  2.977

FIRST 10 POINTS ON THE  2 TH PATTERN
```

```
  2.4109  2.7330  3.0120  3.2373  3.4001  3.4942  3.5161  3.4653  3.3446  3.1597

FIRST 10 POINTS ON THE  3 TH PATTERN
  1.9397  2.1908  2.4135  2.6013  2.7488  2.8520  2.9081  2.9160  2.8760  2.7901

FIRST 10 POINTS ON THE  4 TH PATTERN
  2.3337  2.5976  2.8257  3.0106  3.1459  3.2272  3.2516  3.2186  3.1295  2.9881

FIRST 10 POINTS ON THE  5 TH PATTERN
  1.9935  2.2516  2.4804  2.6734  2.8250  2.9311  2.9888  2.9969  2.9558  2.8674

FIRST 10 POINTS ON THE  6 TH PATTERN
  2.4089  2.6813  2.9168  3.1076  3.2473  3.3312  3.3564  3.3223  3.2304  3.0844
```

X. PLOTS OF SPECIFIED AND SYNTHESIZED PATTERNS

The program of this section plots the synthesized patterns stored on direct access data set 6 by the program of section IX of this report.

The activity on data sets 1 (punched card input) and 6 (direct access input and output) is as follows.

```
          READ(1,10) N6, NG, NP, M, SCL
    10    FORMAT(4I3, E14.7)
          READ(1,12), (PHI(I), I=1, M)
    12    FORMAT(10F8.4)
          READ(1,12)(H(I), I=1, M)
          J9 = NP*NG
          REWIND 6
          IF(N6) 16, 16, 17
    17    DO 18 J=1, N6
          READ(6)
    18    CONTINUE
    16    READ(6)(TFM(I), I=1, J9)
```

The specified pattern evaluated at M angles PHI in degrees is read into H. NG synthesized patterns evaluated at NP equally spaced angles ϕ are read into TFM. All patterns are multiplied by the scale factor SCL and then plotted in inches.

Minimum allocations are given by

```
          DIMENSION PHI(M),H(M),XM(M),YM(M),
             TFM(NP*NG), CS(NP), SN(NP), X(NP), Y(NP)
```

DO loop 15 stores the x and y coordinates of the specified pattern in XM and YM respectively. DO loop 21 stores in CS and SN the cosine and sine of the angles at which the synthesized patterns are evaluated.

The index JG of DO loop 20 denotes the JGth synthesized pattern to be plotted. DO loop 22 puts tick marks on the x axis drawn by statement

26. DO loop 23 puts tick marks on the y axis drawn by statement 27. DO loop 24 plots the first 10 points on the specified pattern. The first 10 points are sufficient because the particular specified pattern used is zero at all but the first 9 points. For a general specified pattern, DO loop 24 should be changed to run from J = 1 to J = M. Statement 28 draws straight lines between the x and y coordinates of the synthesized pattern prepared by DO loop 25.

LISTING OF PROGRAM TO PLOT SPECIFIED AND SYNTHESIZED PATTERNS

```
//              (0034,EE,30S,1,,12),'MAUTZ,JOE',REGION=140K
// MSG I, 70 INCHES OF PLOT PAPER IS REQUIRED
// EXEC FORTGCLG
//FORT.SYSIN DD *
      DIMENSION PHI(36),H(36),XM(36),YM(36),TFM(2900),XX(4),YY(4)
      DIMENSION CS(145),SN(145),AREA(400),X(145),Y(145)
      PI=3.141593
      C=PI/180.
      READ(1,10) N6,NG,NP,M,SCL
   10 FORMAT(4I3,E14.7)
      WRITE(3,11) N6,NG,NP,M,SCL
   11 FORMAT('0 N6 NG NP  M',5X,'SCL'/1X,4I3,E14.7)
      READ(1,12)(PHI(I),I=1,M)
   12 FORMAT(10F8.4)
      WRITE(3,13)(PHI(I),I=1,M)
   13 FORMAT('0PHI'/(1X,10F8.4))
      READ(1,12)(H(I),I=1,M)
      WRITE(3,14)(H(I),I=1,M)
   14 FORMAT('0H'/(1X,10F8.4))
      DO 15 J=1,M
      S1=C*PHI(J)
      S2=SCL*H(J)
      XM(J)=6.+S2*COS(S1)
      YM(J)=5.+S2*SIN(S1)
   15 CONTINUE
      J9=NP*NG
      REWIND 6
      IF(N6) 16,16,17
   17 DO 18 J=1,N6
      READ(6)
   18 CONTINUE
   16 READ(6)(TFM(I),I=1,J9)
      WRITE(3,19)(TFM(I),I=1,10)
   19 FORMAT('0TFM'/(1X,10F8.4))
      XX(1)=1.
      XX(2)=11.
      YY(1)=5.
      YY(2)=5.
      XX(3)=6.
      XX(4)=6.
      YY(3)=0.
      YY(4)=10.
      FN=NP-1
      DEL=2.*PI/FN
      DO 21 J=1,NP
      S1=(J-1)*DEL
      CS(J)=COS(S1)
      SN(J)=SIN(S1)
   21 CONTINUE
      CALL PLOTID
      CALL PLOTS(AREA,400)
      J1=0
      DO 20 JG=1,NG
   26 CALL LINE(XX(1),YY(1),2,1,0,0)
      S1=11.
      DO 22 J=1,11
      CALL SYMBOL(S1,5.,.14,13,0.,-1)
      S1=S1-1.
```

```
   22 CONTINUE
   27 CALL LINE(XX(3),YY(3),2,1,0,0)
      S1=10.
      DO 23 J=1,11
      CALL SYMBOL(6.,S1,.14,13,90.,-1)
      S1=S1-1.
   23 CONTINUE
      DO 24 J=1,10
      CALL SYMBOL(XM(J),YM(J),.14,4,0.,-1)
   24 CONTINUE
      DO 25 J=1,NP
      J1=J1+1
      S1=SCL*TFM(J1)
      X(J)=6.+S1*CS(J)
      Y(J)=5.+S1*SN(J)
   25 CONTINUE
   28 CALL LINE(X,Y,NP,1,0,0)
      CALL PLOT(11.,0.,-3)
   20 CONTINUE
      CALL PLOT(3.,0.,-3)
      STOP
      END
/*
//GO.FT06F001 DD DSNAME=EE0034.REV1,DISP=OLD,UNIT=3330,                  X
//               VOLUME=SER=SU0009,DCB=(RECFM=VS,BLKSIZE=2596,LRECL=2592,X
//               BUFNO=1)
//GO.SYSIN DD *
  0   6145 36 0.5000000E+00
  5.0000 15.0000 25.0000 35.0000 45.0000 55.0000 65.0000 75.0000 85.0000 95.0000
105.0000115.0000125.0000135.0000145.0000155.0000165.0000175.0000185.0000195.0000
205.0000215.0000225.0000235.0000245.0000255.0000265.0000275.0000285.0000295.0000
305.0000315.0000325.0000335.0000345.0000355.0000
  5.0191   3.8637   2.3662   1.7434   1.4142   1.2208   1.1034   1.0353   1.0038   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
/*
//
```

PRINTED OUTPUT

```
 N6 NG NP   M      SCL
  0   6145 36 0.5000000E+00

PHI
  5.0000 15.0000 25.0000 35.0000 45.0000 55.0000 65.0000 75.0000 85.0000 95.0000
105.0000115.0000125.0000135.0000145.0000155.0000165.0000175.0000185.0000195.0000
205.0000215.0000225.0000235.0000245.0000255.0000265.0000275.0000285.0000295.0000
305.0000315.0000325.0000335.0000345.0000355.0000

H
  5.0191   3.8637   2.3662   1.7434   1.4142   1.2208   1.1034   1.0353   1.0038   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000

TFM
  1.9967   2.2996   2.5691   2.7961   2.9730   3.0940   3.1553   3.1555   3.0953   2.9777
```

REFERENCES

[1] J. R. Mautz and R. F. Harrington, "Computational Methods for Pattern Synthesis," Scientific Report No. 2 on Contract F19628-73-C-0047 between Air Force Cambridge Research Laboratory and Syracuse University.

[2] IBM System/360 Scientific Subroutine Package (360A-CM-03X) Version III, Programmer's manual.

[3] C. Fröberg, "Introduction to Numerical Analysis," Addison-Wesley, 1965, pp. 109-112.